Errata:

Page 9, line 4: Replace “α · w_1 and α · w_1” with “α · w_1 and β · w_1”
Page 18, line 6: “second polynomial” should be “first polynomial”
Page 28, line 4: Replace “last” with “first”
Page 39, line −7: Replace “V(I) ⊂ C^n” with “V(I) = {a ∈ C^n : f(a) = 0 for all f ∈ I}”
Page 45, part c of Exercise 9: Delete and replace with “Show that 1 = ∑_j (1/p_j(a_j))p_j.”
Page 46, Exercise 11: Add a new part c of the exercise as follows:
c. For readers familiar with the Hermitian inner product ⟨z, w⟩ = ∑_{i=1}^n z_iw_i for z, w ∈ C^n, let
   \[ h_i(x) = \prod_{j \neq i} (x - p_i, p_i - p_j). \]
   Show that g_i(x) = h_i(x)/h_i(p_i) satisfies part b.
Page 54, line 5: Replace “L(x − ∑_j c_j x^{α(j)}) = 0” with “L(x^α − ∑_j c_j x^{α(j)}) = 0”
Page 55, line 2 of Exercise 2: Replace “x^α > x_1^α” with “x^α ≥ x_1^α”
Page 55, line 2 of Exercise 3: Replace “let x^α be” with “let x^α = x_1^α \cdots x_n^α be”
Page 65, line 7 after the second display: Replace “i_1 > \cdots > i_l” with “i_1 < \cdots < i_l”
Page 71, lines 4–8 of the proof of Theorem (5.2): Replace “Hence we will only . . . invertible matrix” with “Hence we will only discuss the broad outline of the proof. In the case when I is radical, it is possible to turn the sketch that follows into a rigorous proof.”
Page 75, line −3: Replace “−rem(p_{i-1}(t), p_{i-2}(t), t)” with “−rem(p_{i-2}(t), p_{i-1}(t), t)”
Page 75, line −2: Replace “division of p_{i-1} by p_{i-2}” with “division of p_{i-2} by p_{i-1}”
Page 92, line 8: Replace “degrees d_1, . . . , d_n” with “d_0, . . . , d_n”
Page 97, lines 21 and line 24: Replace “Theorem (2.6)” with “Proposition (4.7)”
Page 100, line −2: Replace “Theorem (2.6)” with “Proposition (4.7)”
Page 101, part b of Exercise 10, line 2: Replace “Theorem (2.6)” with “Proposition (4.7)”
Page 102, part c of Exercise 11, line 2: Replace “multiplication by (−1)^n” with “multiplication by (−1)^{n−1}”
Page 102, part d of Exercise 11: Replace “Theorem (3.5)” with “Theorem (3.4)”
Page 106, line 2 of Exercise 8: Replace “total degree 420” with “total degree 210”
Page 107, line 2 of the proof of Proposition (4.7): Replace “(n − 1)! ways” with “n! ways”
Page 108, line 2 of Exercise 11: Replace “D_3” with “D_2”
Page 109, line 3: Replace “Exercise 10” with “Exercise 11”

Page 113, part d of Exercise 22: Replace part d with “Use part c to show that the determinant in (2.8) vanishes whenever \( F_0 = F_1 = F_2 = 0 \) has a nontrivial solution.”

Page 120, line 6 after display (5.12): Replace “\( u_1 = \cdots = u_n = 0 \)” with “\( u_1 = \cdots = u_{n-1} = 0 \)”

Page 123, line 11: Replace “\( A = \mathbb{C}(u)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_n \rangle \)” with “\( A = \mathbb{C}(u_0)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_{n-1} \rangle \)”

Page 123, line 19: Replace “\( \hat{A} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_n \rangle \)” with “\( \hat{A} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_{n-1} \rangle \)”

Page 125, last display: Replace “\( F_0 = \cdots = F_n = 0 \)” with “\( F_1 = \cdots = F_n = 0 \)”

Page 126, line 8: Replace “\( f_0 = \cdots = f_n = 0 \)” with “\( f_1 = \cdots = f_n = 0 \)”

Page 129, line –2: Replace “\( x^\alpha / x_i^d \) has degree \( \leq d - d_i \)” with “\( x^\beta / x_i^d \) has degree \( \leq d - d_i \)”

Page 131, lines 1 and 2 following second-to-last display: Replace “Exercise 12 of Chapter 2, §4” with “Exercise 12 of Chapter 2, §2”

Page 134, line 2 of Exercise 3: Replace “\( (u_0, u_1, u_2, u_3) = (0, 1, 0, 0) \)” with “\( (u_0, u_1, u_2) = (0, 1, 0) \)”

Page 137, line 14: Replace “both these types” with “both types”

Page 138, line –12: Replace “if \( N \neq M \) is an ideal in \( R \) with \( M \subset N \subset R \)” with “if \( N \not\subset M \) is an ideal in \( R \)”

Page 138, lines –10 to –8: Replace “Therefore \( M \ldots \) contained in \( M \)” with “Therefore, every proper ideal of \( R \) is contained in \( M \). Hence \( M \) is maximal and is the only maximal ideal of \( R \)”

Page 142, line 3: Replace “\( \sum_{n \geq 0} f_n(x) \)” with “\( \sum_{m \geq 0} f_m(x) \)”

Page 143, line 4: Replace “\( f_n(x) = \sum_{\alpha \in \mathbb{Z}_{\geq 0}^n, |\alpha| = n} c_\alpha x^\alpha \)” with “\( f_m(x) = \sum_{\alpha \in \mathbb{Z}_{\geq 0}^n, |\alpha| = m} c_\alpha x^\alpha \)”

Page 143, line 5: Replace the display with \( h_m = f_mg_0 + f_{m-1}g_1 + \cdots + f_0g_m \).

Page 147, part a of Exercise 2: Replace “\( \nabla(x^2 - 2x + y^2, x^2 - 4x + 4y^2) \)” with “\( \nabla(x^2 - 2x + y^2, x^2 - 4x + 4y^2) \)”

Page 147, part b of Exercise 2: Replace “\( \langle x^2 - 2x + y^2, x^2 - 4x + 4y^2 \rangle \)” with “\( \langle x^2 - 2x + y^2, x^2 - 4x + 4y^2 \rangle \)”

Page 149, line –18: Replace “note that that” with “note that the”

Page 152, line 6: Replace “Proposition (5.9)” with “Proposition (5.15)”

Page 152, line 3 of Exercise 3: Replace “\( f_2 = 6y - x^3 + 9x \)” with “\( f_2 = 6y - x^3 + 9x = 0 \)”

Page 152, line 1 of part e of Exercise 3: Replace “\( \text{Res}(f_1, f_2, y) \)” with “\( \text{Res}^y(f_1, f_2) \)” as defined in (5.14) of Chapter 3’
Page 152, line 4 of part e of Exercise 3: Replace “Res(f_1, f_2, x)” with “Res^r(f_1, f_2)”
Page 156, part c of Exercise 10, line 4: Replace “A_i ⇔ f(p) = λ” with “A_i ⇔ f(p_i) = λ”
Page 160, line −2: Replace “ker(M) ∩ Z_{n≥0}” with “ker(M) ∩ Z^n”
Page 161, line −3: Replace “S = \{1 + g : \text{LT}(g) < 1\}” with “S = \{1 + g : g = 0 \text{ or } \text{LT}(g) < 1\}”
Page 163, line 1 of part a of Exercise 5: Replace “let h ∈ A” with “let h ∈ \text{Loc}_>(A)”
Page 163, line 1 of part b of Exercise 5: Replace “Let r ∈ R” with “Let r ∈ \text{Loc}_>(A)”
Page 166, lines −20 and −18: Replace “t^a > t^a x^β” with “t^a > t^a x^β” (twice)
Page 171, line 3 of part c of Exercise 8: Replace “1/(1 + h)” with “1/(1 + g)”
Page 172, line 2 of Exercise 11: Replace “(for local orders)” with “(for degree-anticompatible orders)”
Page 172, line 1 of part a of Exercise 11: Replace “Let > be a local order” with “Let > be a degree-anticompatible order”
Page 193, line −2: Replace “When M and N are free modules,” with “When M = R^l and N = R^m,”
Page 197, line −10: Besides the 1994 paper [PW] by Park and Woodburn, we should also mention two other papers that deal with algorithmic aspects of the Quillen-Suslin result:
Page 199, line 4: “Equivalently, we think” should be “Equivalently, we think”
Page 200, line 18: Replace “one-one” with “one-to-one”
Page 201, line −2 of proof of Proposition (1.11): Replace “\sum c_i m_i” with “\sum c_i f_i”
Page 203, line 1: Replace “Let \varphi : M \to N.” with “Let \varphi : M \to N be an R-module homomorphism.”
Page 203, line 2 of part a of Exercise 23: Replace “\{a f : a ∈ I, f ∈ M\}” with “\{\sum_{i=1}^\ell a_i f_i : a_i ∈ I, f_i ∈ M \text{ for } i = 1, \ldots, \ell\}”.
Page 203, line −16: Replace “We let R = k[x, y]” with “Let R = k[x, y], where k is a field of characteristic different from 2,”
Page 203, lines −9 to −1: Delete and replace with the following:

a. Verify that f = (f_1, f_2, f_3)^T = (1, −x/2, −1/2)^T ∈ R^3 satisfies (1 + x)f_1 + (1 − y)f_2 + (x + xy)f_3 = 1.
b. Let \( I \) be the \( 3 \times 3 \) identity matrix. Verify that the columns \( g_1, g_2, g_3 \) of the matrix \( I - f \cdot A \) span \( \ker A \). Hint: If \( A\tilde{f} = 0 \), then \( \tilde{f} = (I - f \cdot A)\tilde{f} \) is a linear combination of the columns of \( I - f \cdot A \).

c. Show that \( \{g_1, g_2\} \) is a basis of \( \ker A \). (Unfortunately, the result of part c is special to the choice of \( f \) made in part a. If \( f \) is an arbitrary solution of \( Af = 1 \), then the first two columns of \( I - f \cdot A \) need not give a basis of the kernel.)
Page 239, line 1: Replace “matrix of $M/mM.$” with “matrix of $M/mM$?”
Page 239, line 10: Replace “columns of $M$” with “columns of $A$”
Page 239, line 13: Replace “in $P/IP$” with “in $M/IM$”
Page 240, line 4: Replace “have have” with “have”
Page 242, line 14: Replace “$m \times 1$ matrix” with “$r \times 1$ matrix”
Page 242, Proposition (4.11): Replace “$Q$ be a local ring, $M$ a finitely generated $Q$-” with “$R$ be a local ring, $M$ a finitely generated $R$-”
Page 243, line −16: Replace “$M/mM$ Since” with “$M/mM.$ Since”
Page 245, part c of Exercise 10: Replace “$0 = F_0(M) \subset F_1(M) \subset \cdots \subset F_{s+1} = R$” with “$0 = F_{-1}(M) \subset F_0(M) \subset \cdots \subset F_s = R$”
Page 248, line −9: Replace “Exercise 12” with “Exercise 28”
Page 253, line −6: Replace with “$M = \langle yz - xw, y^3 - x^2 z, xz^2 - y^2 w, z^3 - yw^2 \rangle$”
Page 254, line 1: Replace with “$M = \text{ideal}(yz - xw, y^3 - x^2 z, xz^2 - y^2 w, z^3 - yw^2)$”
Page 263, line 1 of (3.3) Proposition: Replace “be submodule” with “be a submodule”
Page 269, line 1 of Exercise 3: Replace “finitely generated” with “finitely generated graded”
Page 270, line 6: Replace with “$M = \langle yz - xw, y^3 - x^2 z, xz^2 - y^2 w, z^3 - yw^2 \rangle$”
Page 270, line 9: Replace with “$R(-2) \oplus R(-3)^3 \rightarrow R$”
Page 275, line 2: Replace “$F_{\ell+2} \xrightarrow{\phi_{\ell+1}} F_{\ell+1}$” with “$F_{\ell+2} \xrightarrow{\phi_{\ell+1}} F_{\ell+1}$”
Page 275, line 7: Replace “$+ c_2 \phi_{\ell-1}(u_m)$” with “$+ c_2 \phi_{\ell-1}(u_m)$”
Page 275, line 9: Replace “$i = 2, \ldots, m$” with “$i = 2, \ldots, t$”
Page 279, line 2 of Exercise 14: Replace “$\psi : G_\ell \rightarrow G_{\ell-1}$” with “$\psi_\ell : G_\ell \rightarrow G_{\ell-1}$”
Page 279, lines 6-7 of Exercise 14: Replace “$A_{01} = (c_2, \ldots, c_\ell)$ as in (3.16), and $A_{10} = (d_2, \ldots, d_m)^T$” with “$A_{10} = (c_2, \ldots, c_\ell)^T$ as in (3.16), and $A_{01} = (d_2, \ldots, d_m)$”
Page 279, line 10 of Exercise 14: Replace “$B_\ell = A_{00} - A_{01}A_{11}^{-1}A_{10}$” with “$B_\ell = A_{11} - A_{10}A_{00}^{-1}A_{01}$”
Page 279, line 11 of Exercise 14: Replace “What’s remarkable is that this formula is identical to” with “This formula is a slight variation of the formula in”
Page 289, line 3 of Definition (4.16): “to the minimal” should be “to be the minimal”

Page 290, line 12: Replace “for $S/J$ to” with “for $R/J$ to”

Page 293, line 3: Replace “$\bar{c} = p_1q_2 - p_1q_1$” with “$\bar{c} = p_1q_2 - p_2q_1$”

Page 293, line 19: Replace “GCD($a_1, \ldots, a_m$) = 1” with “GCD($a_1, \ldots, a_m, c$) = 1”

Page 297, part a of Exercise 12, line 3: Replace “$R$” with “$S$”

Page 297, line 15: Replace “$R$” with “$S$”

Page 303, part d of Exercise 25: Replace “parts b, c and d” with “parts b and d”

Page 308, line before Exercise 4: Add a new sentence “We also regard $Q$ as a face of itself.”

Page 308, line following Exercise 4: Replace “Every face” with “Every proper face”

Page 314, line 3 of Exercise 1: Replace “You already did a special case of this in Exercise 2 of Chapter 3, §2” with “This is a special case of Exercise 2 of Chapter 3, §2”

Page 314, the last row of the matrix in display (2.4): Replace “$c_0 - x$” with “$c_0 - z$”

Page 314, part a of Exercise 2, line 4: Replace “$st^2$” with “$s^2t$”

Page 319, line 1 of Exercise 6: Replace “Then” with “Use the bracket notation introduced in Theorem (3.5) of Chapter 3, §3 to”

Page 325, display (3.9): Replace “$F(x_1, \ldots, x_N)$” with “$F(x_1, \ldots, x_N)$”

Page 325, line 1 of proof of Lemma (3.10): Replace “$m = \sum_{i=1}^{n} a_i e_i$” with “$m = \sum_{i=1}^{n} b_i e_i$”

Page 325, line 2 of Exercise 4: Replace “Exercise 3” with “Exercise 7 of §1”

Page 327, line 9: In the statement of Theorem (3.13), replace “$A = \{m_1, \ldots, m_l\} \subset \mathbb{Z}_{\geq 0}^n$” with “$A = \{m_1, \ldots, m_l\} \subset \mathbb{Z}^n$”

Page 328, line 11: In two places, replace “$x_0, \ldots, x_N$” with “$x_1, \ldots, x_N$”

Page 331, part d of Exercise 11, line 2: Replace “$x_1, \ldots, x_n$” with “$x_1, \ldots, x_N$”

Page 331, part d of Exercise 11, line 3: Replace “$x_1, \ldots, x_n$” with “$x_1, \ldots, x_N$”

Page 334, line 11: Replace “which is the called” with “which is called”

Page 334, line 10: Replace “If $S$ is subset of” with “If $S$ is a subset of”

Page 339, line 12: Replace “part b” with “part c”

Page 339, line 18: Replace “Exercise 5” with “Exercise 6”

Page 342, line 1: Replace “$\mu \cdot a_Q(\nu) \geq 0$” with “$\mu \cdot a_Q(\nu)/\|\nu\| \geq 0$”

Page 343, 7 lines below display (5.2): Replace “equivalent” with “equivalent to”

Page 352, part e of Exercise 4, line 1: Replace “$d \mapsto d/t$” with “$d \mapsto d/t^8$”

Page 359, line 7: Replace “polyedral” with “polyhedral”

Page 359, line 4 of Definition (6.4): Replace “is a face” with “is either empty or a face”

Page 365, Figure 7.9: The figure is wrong. Here is the correct figure.
Page 373, line 3: Replace “Chapter 2” with “Chapter 3”

Page 416, part b of Exercise 4, line 6: Replace “$g_3 = (2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)$” with “$g_3 = (2xy^2 + y^3, x^2y + 2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)$”

Page 417, part c of Exercise 5, last line: Replace “if $k \geq 3$” with “if $k \geq 4$”

Page 422, part d of Exercise 8: Replace “$M(\Delta', r)$” with “$M(\Delta, r)$”

Page 423, line 7: Replace “expression (3.19)” with “expression (3.18)”

Page 425, Exercise 14, line 2: Replace “hereditary complex” with “hereditary simplicial complex”

Page 431, line 6: The left-hand side of the equation should be “$\{x^2 - y, yz + xz - y^2\}$”

Page 433, line 1: Replace “that $w$” with “that $w$”

Page 438, line 18: Replace “is the positive orthant” with “in the positive orthant”

Page 440, third display: Replace “$\langle \mathrm{in}_{\text{new}}(G_{\text{old}}) \rangle$” with “$\langle \mathrm{LT}_{\text{new}}(\langle \mathrm{in}_{\text{new}}(G_{\text{old}}) \rangle) \rangle$”

Page 440, line 9: In two places, replace “$q_{j,g}$” with “$p_{j,g}$”

Page 444, line 10: “$\mathbf{w}_t \cdot v_1 = 6$” should be “$\mathbf{w}_t \cdot v_1 = 11$”

Page 444, line 14: “$v_2 = (0, -1, -1)$” should be “$v_3 = (0, 1, -1)$”

Page 473, line 3: Replace “$\langle x_1^{n_1-1}, \ldots, x_m^{n_m-1} - 1 \rangle$” with “$\langle x_1^{n_1-1}, \ldots, x_m^{n_m-1} \rangle$”

Page 474, line 6: Replace “$\langle x_1^{n_1-1}, \ldots, x_m^{n_m-1} - 1 \rangle$” with “$\langle x_1^{n_1-1}, \ldots, x_m^{n_m-1} \rangle$”

Page 496, line 7: Replace “of elements” with “of nonzero elements”

Page 496, line 3: Replace “are verified” with “are satisfied”

Page 502, line 10: Replace “$x_1x_2^3$” with “$x_1x_2^3$”

Page 553, first column, line 14: Replace “Faugère, C.” with “Faugère, J.-C.”