Using Algebraic Geometry, second edition

September 15, 2021

Errata:

Page 9, line 4: Replace “$\alpha \cdot w_1$ and $\alpha \cdot w_1$” with “$\alpha \cdot w_1$ and $\beta \cdot w_1$”

Page 18, line 6: “second polynomial” should be “first polynomial”

Page 28, line 4: Replace “last” with “first”

Page 39, line $-7$: Replace “$V(I) \subset \mathbb{C}^n$” with “$V(I) = \{a \in \mathbb{C}^n : f(a) = 0 \text{ for all } f \in I\}$”

Page 45, part c of Exercise 9: Delete and replace with “Show that $1 = \sum j (1/p_j(a_j))p_j$.”

Page 46, Exercise 11: Add a new part c of the exercise as follows:

c. For readers familiar with the Hermitian inner product $\langle z, w \rangle = \sum_{i=1}^{n} z_i \bar{w}_i$ for $z, w \in \mathbb{C}^n$, let

$$h_i(x) = \prod_{j \neq i} \langle x - p_i, p_i - p_j \rangle.$$ 

Show that $g_i(x) = h_i(x)/h_i(p_i)$ satisfies part b.

Page 54, line 5: Replace “$L(x - \sum_j c_j x^\alpha(j)) = 0$” with “$L(x^\alpha - \sum_j c_j x^\alpha(j)) = 0$”

Page 55, line 2 of Exercise 2: Replace “$x^\alpha > x_1^\alpha$” with “$x^\alpha \geq x_1^\alpha$”

Page 55, line 2 of Exercise 3: Replace “let $x^\alpha$ be” with “let $x^\alpha = x_1^\alpha \cdots x_n^\alpha$ be”

Page 65, line 7 after the second display: Replace “$i_1 > \cdots > i_l$” with “$i_1 < \cdots < i_l$”

Page 71, lines 4–8 of the proof of Theorem (5.2): Replace “Hence we will only … invertible matrix” with “Hence we will only discuss the broad outline of the proof. In the case when $I$ is radical, it is possible to turn the sketch that follows into a rigorous proof.”

Page 75, line $-3$: Replace “$-\text{rem}(p_{i-1}(t), p_{i-2}(t), t)$” with “$-\text{rem}(p_{i-2}(t), p_{i-1}(t), t)$”

Page 75, line $-2$: Replace “division of $p_{i-1}$ by $p_{i-2}$” with “division of $p_{i-2}$ by $p_{i-1}$”

Page 82, line 2 of Exercise 7: Replace “$\deg(r) < \deg(g)$” with “$\deg(r) < \deg(f)$”

Page 92, line 8: Replace “degrees $d_1, \ldots, d_n$” with “$d_0, \ldots, d_n$”

Page 97, lines 21 and line 24: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 100, line $-2$: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 101, part b of Exercise 10, line 2: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 102, part c of Exercise 11, line 2: Replace “multiplication by $(-1)^n$” with “multiplication by $(-1)^{n-1}$”

Page 102, part d of Exercise 11: Replace “Theorem (3.5)” with “Theorem (3.4)”

Page 106, line 2 of Exercise 8: Replace “total degree 420” with “total degree 210”

Page 107, line 2 of the proof of Proposition (4.7): Replace “$(n - 1)!$ ways” with “$n!$ ways”
Page 108, line 2 of Exercise 11: Replace “$D'_3$” with “$D'_2$”

Page 109, line 3: Replace “Exercise 10” with “Exercise 11”

Page 113, part d of Exercise 22: Replace part d with “Use part c to show that the determinant in (2.8) vanishes whenever $F_0 = F_1 = F_2 = 0$ has a nontrivial solution.”

Page 120, line 6 after display (5.12): Replace “$u_1 = \cdots = u_n = 0$” with “$u_1 = \cdots = u_{n-1} = 0$”

Page 123, line 11: Replace “$A = \mathbb{C}(u)[x_1, \ldots, x_n]/(u - x_n, f_1, \ldots, f_n)$” with “$A = \mathbb{C}(u_0)[x_1, \ldots, x_n]/(u - x_n, f_1, \ldots, f_{n-1})$”

Page 123, line 19: Replace “$\hat{\mathcal{A}} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/(\hat{f}_1, \ldots, \hat{f}_{n-1})$” with “$\hat{\mathcal{A}} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/(\hat{f}_1, \ldots, \hat{f}_{n-1})$”

Page 152, line 3: Replace “Proposition (5.9)” with “Proposition (5.15)”

Page 152, line 6: Replace “Proposition (5.15)” with “Proposition (5.21)”

Page 143, line 4: Replace “$\sum$” with “$\bigoplus$”

Page 142, line 3: Replace “$\mathcal{R}$ ideal of $R$” with “$\mathcal{R}$ every proper ideal of $R$ is an ideal in $R$”

Page 147, part a of Exercise 2: Replace “$\mathbf{V}(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$” with “$\mathbf{V}(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$”

Page 147, part b of Exercise 2: Replace “$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^2 \rangle$” with “$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^2 \rangle$”

Page 149, line −18: Replace “note that that” with “note that the”

Page 152, line 6: Replace “Proposition (5.9)” with “Proposition (5.15)”

Page 152, line 3 of Exercise 3: Replace “$f_2 = 6y - x^3 + 9x$,” with “$f'_2 = 6y - x^3 + 9x = 0$,”
Page 152, line 1 of part e of Exercise 3: Replace “Res\((f_1, f_2, y)\)” with “Res^y(f_1, f_2)” as defined in (5.14) of Chapter 3”

Page 152, line 4 of part e of Exercise 3: Replace “Res\((f_1, f_2, x)\)” with “Res^x(f_1, f_2)”

Page 156, part c of Exercise 10, line 4: Replace “A_i \iff f(p) = \lambda” with “A_i \iff f(p_i) = \lambda”

Page 160, line −2: Replace “ker\((M) \cap \mathbb{Z}_{\geq 0}\)” with “ker\((M) \cap \mathbb{Z}\)”

Page 161, line −3: Replace “S = \{1 + g : \text{LT}(g) < 1\}” with “S = \{1 + g : g = 0 \text{ or } \text{LT}(g) < 1\}”

Page 163, line 1 of part a of Exercise 5: Replace “let h \in A” with “let h \in \text{Loc}_>(A)”

Page 163, line 1 of part b of Exercise 5: Replace “Let r \in R” with “Let r \in \text{Loc}_>(A)”

Page 166, lines −20 and −18: Replace “t^a > t'^a x^\beta” with “t^a > t'^a x^\beta” (twice)

Page 171, line 3 of part c of Exercise 8: Replace “1/(1 + h)” with “1/(1 + g)”

Page 172, line 2 of Exercise 11: Replace “(for local orders)” with “(for degree-anticompatible orders)”

Page 172, line 1 of part a of Exercise 11: Replace “Let > be a local order” with “Let > be a degree-anticompatible order”

Page 193, line −2: Replace “When M and N are free modules,” with “When M = R^l and N = R^m,”

Page 197, line −10: Besides the 1994 paper [PW] by Park and Woodburn, we should also mention two other papers that deal with algorithmic aspects of the Quillen-Suslin result:


Page 199, line 4: “Equivalently, we think” should be “Equivalently, we think”

Page 200, line 18: Replace “one-one” with “one-to-one”

Page 201, line −2 of proof of Proposition (1.11): Replace “\(\sum c_i m_i\)” with “\(\sum c_i f_i\)”

Page 203, line 1: Replace “Let \(\varphi : M \to N\)” with “Let \(\varphi : M \to N\) be an R-module homomorphism.”

Page 203, line 2 of part a of Exercise 23: Replace \(\{af : a \in I, f \in M\}\) with \(\{\sum_{i=1}^\ell a_i f_i : a_i \in I, f_i \in M\ \text{for } i = 1, \ldots, \ell\}\)

Page 203, line −16: Replace “We let \(R = k[x, y]\)” with “Let \(R = k[x, y]\), where \(k\) is a field of characteristic different from 2,”

Page 203, lines −9 to −1: Delete and replace with the following:

a. Verify that \(f = (f_1, f_2, f_3)^T = (1, -x/2, -1/2)^T \in \mathbb{R}^3\) satisfies \((1 + x)f_1 + (1 - y)f_2 + (x + xy)f_3 = 1.\)
b. Let $I$ be the $3 \times 3$ identity matrix. Verify that the columns $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$ of the matrix $I - \mathbf{f} \cdot A$ span $\ker A$. Hint: If $A\mathbf{f} = 0$, then $\mathbf{f} = (I - \mathbf{f} \cdot A)\mathbf{f}$ is a linear combination of the columns of $I - \mathbf{f} \cdot A$.

c. Show that $\{\mathbf{g}_1, \mathbf{g}_2\}$ is a basis of $\ker A$. (Unfortunately, the result of part c is special to the choice of $\mathbf{f}$ made in part a. If $\mathbf{f}$ is an arbitrary solution of $A\mathbf{f} = 1$, then the first two columns of $I - \mathbf{f} \cdot A$ need not give a basis of the kernel.)
Page 237, line -2: Add the sentence “If \( s = 1 \), then \((1 - a_1)f_1 = 0\). This implies \( f_1 = 0\), which contradicts \( M \neq 0\).

Page 239, line 1: Replace “matrix of \( M/\mathfrak{m}M\)” with “matrix of \( M/\mathfrak{m}M\)?”

Page 239, line 10: Replace “columns of \( M\)” with “columns of \( A\)”

Page 239, line 13: Replace “in \( P/IP\)” with “in \( M/IM\)”

Page 240, line 4: Replace “have have” with “have”

Page 242, line 14: Replace “\( m \times 1\) matrix” with “\( r \times 1\) matrix”

Page 242, Proposition (4.11): Replace “\( Q\) be a local ring, \( M\) a finitely generated \( Q\)-” with “\( R\) be a local ring, \( M\) a finitely generated \( R\)-”

Page 243, line -16: Replace “\( M/\mathfrak{m}M\) Since” with “\( M/\mathfrak{m}M\). Since”

Page 245, part c of Exercise 10: Replace “0 = \( F_0(M) \subset F_1(M) \subset \cdots \subset F_{s+1} = R\)” with “0 = \( F_{-1}(M) \subset F_0(M) \subset \cdots \subset F_s = R\)”

Page 246, line 1: Replace “Exercise 12” with “Exercise 28”

Page 248, line -9: Replace “Exercise 12” with “Exercise 28”

Page 253, line -6: Replace with “\( M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle\)”

Page 254, line 1: Replace with “\( \mathcal{M} = \text{ideal}(y*z-x*w,y^3-x^2*z,x*z^2-y^2*w,z^3-y*w^2)\)”

Page 256, line after second display: Replace “im(\( \varphi_2 \)) = Syz(\( G_1 \))” with “im(\( \varphi_2 \)) = Syz(\( G_0 \)) = ker(\( \varphi_1 \)) in \( F_1\)”

Page 256, lines 1 and 2 after second display: Replace “obtain \( \varphi_i : F_i \to F_{i-1}\), where im(\( \varphi_i \)) = Syz(\( G_{i-1} \)) and \( G_i \subset R^{r_i}\) is a Gröbner” with “obtain \( \varphi_{i+1} : F_{i+1} \to F_i\), where im(\( \varphi_{i+1} \)) = Syz(\( G_{i-1} \)) = ker(\( \varphi_i \)) in \( F_i\) and \( G_i \subset F_i = R^{r_i}\) is a reduced Gröbner”

Page 256, lines 2 and 3 above display (2.5): Replace “the leading terms of the reduced Gröbner basis \( \mathcal{G}_\ell\)” with “the reduced Gröbner basis \( \mathcal{G}_\ell\) of Syz(\( G_{\ell-1} \)) \( \subset F_\ell\) is either empty or its leading terms”

Page 256, display (2.5): Add \( \varphi_{\ell-1}\) above the second arrow and put a period at the end of the display.

Page 256, line after display (2.5): Replace “and the leading” with “When \( \mathcal{G}_\ell = \emptyset\), ker(\( \varphi_\ell \)) = \{0\} and \( \varphi_\ell\) is injective. Thus we can extend (2.5) to a free resolution of length \( \ell \leq n\) by adding a zero at the left. Otherwise, the leading”

Page 256, three lines below display (2.5): Replace “Syz(\( G_{\ell-1} \)) is a free module” with “\( R^n/ker(\varphi_\ell) \simeq \text{im}(\varphi_\ell) = \text{ker}(\varphi_{\ell-1})\) is a free module”

Page 256, four lines below display (2.5): Replace “we can extend (2.5)” should be “we can replace \( F_\ell\) with the free module ker(\( \varphi_{\ell-1} \)) and extend (2.5)”

Page 263, line 1: Replace “from (1.8)” with “from (1.7)”

Page 263, line 6: Replace “see (1.16)” with “(see (1.14))”

Page 264, Exercise 8: Add the following new part d:
d. Show that $R^t/M$ is also a free module. Hint: Let $N \subset R^t$ be the free submodule generated by the standard basis vectors that are not leading terms of elements of $G$. Use the division algorithm with respect to $G$ to show that the induced map $N \rightarrow R^t/M$ is an isomorphism.

Page 265, part a of Exercise 11: Replace “of the the” with “of the”
Page 265, part b of Exercise 11, line 2: Replace “$(-1) \det(A_i)$, where $A_i$” with “$(-1) \det(A_i)$, where $A_i$”
Page 265, part d of Exercise 11, line -1: Replace “$= pB$ for some $B \in R^m$” with “$= pC$ for some $C \in R^m$”
Page 267, line 1 of (3.3) Proposition: Replace “be submodule” with “be a submodule”
Page 269, line 1 of Exercise 3: Replace “finitely generated” with “finitely generated graded”
Page 270, line 6: Replace with “$M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle$”
Page 270, line 9: Replace with “$R(-2) \oplus R(-3)^3 \rightarrow R$”
Page 275, line 2: Replace “$F_{t+2} \xrightarrow{\varphi_{t+1}} F_{t+1}$” with “$F_{t+2} \xrightarrow{\varphi_{t+2}} F_{t+1}$”
Page 275, line 7: Replace “$+ c_2 \varphi_{t-1}(u_m)$” with “$+ c_t \varphi_{t-1}(u_t)$”
Page 275, line 9: Replace “$i = 2, \ldots, m$” with “$i = 2, \ldots, t$”
Page 279, line 2 of Exercise 14: Replace “$\psi : G_\ell \rightarrow G_{\ell-1}$” with “$\psi_\ell : G_\ell \rightarrow G_{\ell-1}$”
Page 279, lines 6-7 of Exercise 14: Replace “$A_{01} = (c_2, \ldots, c_t)$ as in (3.16), and $A_{10} = (d_2, \ldots, d_m)^T$” with “$A_{10} = (c_2, \ldots, c_t)^T$ as in (3.16), and $A_{01} = (d_2, \ldots, d_m)$”
Page 279, line 10 of Exercise 14: Replace “$B_\ell = A_{00} - A_{01}A_{11}^{-1}A_{10}$” with “$B_\ell = A_{11} - A_{10}A_{00}^{-1}A_{01}$”
Page 279, line 11 of Exercise 14: Replace “What’s remarkable is that this formula is identical to” with “This formula is a slight variation of the formula in”
Page 289, line 3 of Definition (4.16): “to the minimal” should be “to be the minimal”
Page 290, line 12: Replace “for $S/J$ to” with “for $R/J$ to”
Page 293, line 3: Replace “$\bar{c} = p_1q_2 - p_1q_2$” with “$\bar{c} = p_1q_2 - p_2q_1$”
Page 293, line -19: Replace “GCD$(a_1, \ldots, a_m) = 1$” with “GCD$(a_1, \ldots, a_m, c) = 1$”
Page 297, part a of Exercise 12, line 3: Replace “$R^G$” with “$S^G$”
Page 297, line -3: Replace “$R^G$” with “$S^G$”
Page 303, part d of Exercise 25: Replace “parts b, c and d” with “parts b and d”
Page 308, line before Exercise 4: Add a new sentence “We also regard $Q$ as a face of itself.”
Page 308, line following Exercise 4: Replace “Every face” with “Every proper face”
Page 314, line 3 of Exercise 1: Replace “You already did a special case of this in Exercise 2 of Chapter 3, §2” with “This is a special case of Exercise 2 of Chapter 3, §2”
Page 314, the last row of the matrix in display (2.4): Replace “$c_0 - x$” with “$c_0 - z$”
Page 314, part a of Exercise 2, line 4: Replace “$st^2$” with “$s^2t$”
Page 319, line 1 of Exercise 6: Replace “Then” with “Use the bracket notation introduced in Theorem (3.5) of Chapter 3, §3 to”
Page 325, display (3.9): Replace “$F(x_1, \ldots, x_n)$” with “$F(x_1, \ldots, x_N)$”
Page 325, line 1 of proof of Lemma (3.10): Replace “$m = \sum_{i=1}^{n} a_i e_i$” with “$m = \sum_{i=1}^{n} b_i e_i$”
Page 325, line 2 of Exercise 4: Replace “Exercise 3” with “Exercise 7 of §1”
Page 327, line 9: In the statement of Theorem (3.13), replace “$\mathcal{A} = \{m_1, \ldots, m_l\} \subset \mathbb{Z}_{\geq 0}^n$” with “$\mathcal{A} = \{m_1, \ldots, m_l\} \subset \mathbb{Z}^n$”
Page 328, line 11: In two places, replace “$x_0, \ldots, x_N$” with “$x_1, \ldots, x_N$”
Page 331, part d of Exercise 11, line 2: Replace “$x_1, \ldots, x_n$” with “$x_1, \ldots, x_N$”
Page 331, part d of Exercise 11, line 3: Replace “$x_1, \ldots, x_n$” with “$x_1, \ldots, x_N$”
Page 334, line −11: Replace “which is the called” with “which is called”
Page 334, line −10: Replace “If $S$ is subset of” with “If $S$ is a subset of”
Page 339, line 12: Replace “part b” with “part c”
Page 339, line 18: Replace “Exercise 5” with “Exercise 6”
Page 342, line 1: Replace “$\mu \cdot a_Q(\nu) \geq 0$” with “$\mu \cdot a_Q(\nu)/\|\nu\| \geq 0$”
Page 343, 7 lines below display (5.2): Replace “equivalent” with “equivalent to”
Page 352, part e of Exercise 4, line 1: Replace “$d \mapsto d/t$” with “$d \mapsto d/t^8$”
Page 359, line -7: Replace “polyedral” with “polyhedral”
Page 359, line 4 of Definition (6.4): Replace “is a face” with “is either empty or a face”
Page 365, Figure 7.9: The figure is wrong. Here is the correct figure.

Page 373, line 3: Replace “Chapter 2” with “Chapter 3”
Page 416, part b of Exercise 4, line 6: Replace “\(g_3 = (2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)\)” with \(g_3 = (2xy^2 + y^3, x^2y + 2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)\)”

Page 417, part c of Exercise 5, last line: Replace “if \(k \geq 3\)” with “if \(k \geq 4\)”

Page 422, part d of Exercise 8: Replace “\(M(\Delta', r)\)” with “\(M(\overline{\Delta}, r)\)”

Page 423, line 7: Replace “expression (3.19)” with “expression (3.18)”

Page 425, Exercise 14, line 2: Replace “hereditary complex” with “hereditary simplicial complex”

Page 431, line -6: The left-hand side of the equation should be \(\{\overline{x}^2 - y, yz + xz - y^2\}\)

Page 433, line 1: Replace “that \(w\)” with “that \(w\)”

Page 438, line -18: Replace “is the positive orthant” with “in the positive orthant”

Page 440, third display: Replace “\((\text{in}_{\text{new}}(G_{\text{old}}))\)” with “\((\text{LT}_{\text{new}}(\langle \text{in}_{\text{new}}(G_{\text{old}}) \rangle))\)”

Page 440, line -9: In two places, replace “\(q_{j,g}\)” with “\(p_{j,g}\)”

Page 444, line 10: “\(w_t \cdot v_1 = 6\)” should be “\(w_t \cdot v_1 = 11\)”

Page 444, line 14: “\(v_2 = (0, -, -1)\)” should be “\(v_3 = (0, 1, -1)\)”

Page 473, line -3: Replace “\(\langle x_1^{n_1-1} - 1, \ldots, x_m^{n_m-1} - 1 \rangle\)” with “\(\langle x_1^{n_1} - 1, \ldots, x_m^{n_m} - 1 \rangle\)”

Page 474, line 6: Replace “\(\langle x_1^{n_1-1} - 1, \ldots, x_m^{n_m-1} - 1 \rangle\)” with “\(\langle x_1^{n_1} - 1, \ldots, x_m^{n_m} - 1 \rangle\)”

Page 496, line 7: Replace “of elements” with “of nonzero elements”

Page 496, line -3: Replace “are verified” with “are satisfied”

Page 502, line 10: Replace “\(x_1x_2^4\)” with “\(x_1x_2^3\)”

Page 553, first column, line -14: Replace “Faugère, C.” with “Faugère, J.-C.”