Using Algebraic Geometry, second edition

September 15, 2021

Errata:

Page 9, line 4: Replace “$\alpha \cdot w_1$ and $\alpha \cdot w_1$” with “$\alpha \cdot w_1$ and $\beta \cdot w_1$”
Page 18, line 6: “second polynomial” should be “first polynomial”
Page 28, line 4: Replace “last” with “first”
Page 39, line $-7$: Replace “$V(I) \subset \mathbb{C}^n$” with “$V(I) = \{a \in \mathbb{C}^n : f(a) = 0$ for all $f \in I\}$”
Page 45, part c of Exercise 9: Delete and replace with “Show that $1 = \sum_j (1/p_j(a_j))p_j.$”
Page 46, Exercise 11: Add a new part c of the exercise as follows:

\[ h_i(x) = \prod_{j \neq i} \langle x - p_i, p_i - p_j \rangle. \]

Show that $g_i(x) = h_i(x)/h_i(p_i)$ satisfies part b.

Page 54, line 5: Replace “$L(x - \sum_j c_jx^{\alpha(j)} = 0$” with “$L(x^\alpha - \sum_j c_jx^{\alpha(j)} = 0$”
Page 55, line 2 of Exercise 2: Replace “$x^\alpha > x_1^\alpha$” with “$x^\alpha \geq x_1^\alpha$”
Page 55, line 2 of Exercise 3: Replace “let $x^\alpha$ be” with “let $x^\alpha = x_1^{a_1} \cdots x_n^{a_n}$ be”
Page 65, line 7 after the second display: Replace “$i_1 > \cdots > i_l$” with “$i_1 < \cdots < i_l$”
Page 71, lines 4-8 of the proof of Theorem (5.2): Replace “Hence we will only . . . invertible matrix” with “Hence we will only discuss the broad outline of the proof. In the case when $I$ is radical, it is possible to turn the sketch that follows into a rigorous proof.”
Page 75, line $-3$: Replace “$-\text{rem}(p_{i-1}(t), p_{i-2}(t), t)$” with “$-\text{rem}(p_{i-2}(t), p_{i-1}(t), t)$”
Page 75, line $-2$: Replace “division of $p_{i-1}$ by $p_{i-2}$” with “division of $p_{i-2}$ by $p_{i-1}$”
Page 82, line 2 of Exercise 7: Replace “$\deg(r) < \deg(g)$” with “$\deg(r) < \deg(f)$”
Page 92, line 8: Replace “degrees $d_1, \ldots, d_n$” with “$d_0, \ldots, d_n$”
Page 97, lines 21 and line 24: Replace “Theorem (2.6)” with “Proposition (4.7)”
Page 100, line $-2$: Replace “Theorem (2.6)” with “Proposition (4.7)”
Page 101, part b of Exercise 10, line 2: Replace “Theorem (2.6)” with “Proposition (4.7)”
Page 102, part c of Exercise 11, line 2: Replace “multiplication by $(-1)^n$” with “multiplication by $(-1)^{n-1}$”
Page 102, part d of Exercise 11: Replace “Theorem (3.5)” with “Theorem (3.4)”
Page 106, line 2 of Exercise 8: Replace “total degree 420” with “total degree 210”
Page 107, line 2 of the proof of Proposition (4.7): Replace “$(n-1)!$ ways” with “$n!$ ways”
Page 108, line 2 of Exercise 11: Replace “$D_3'$” with “$D'_2$”

Page 109, line 3: Replace “Exercise 10” with “Exercise 11”

Page 113, part d of Exercise 22: Replace part d with “Use part c to show that the determinant in (2.8) vanishes whenever $F_0 = F_1 = F_2 = 0$ has a nontrivial solution.”

Page 120, line 6 after display (5.12): Replace “$f$” with “$\hat{f}$”

Page 126, line 8: Replace “$f$” with “$\hat{f}$”

Page 131, lines 1 and 2 following second-to-last display: Replace “Exercise 12 of Chapter 2, §4” with “Exercise 12 of Chapter 2, §2”

Page 134, line 2 of Exercise 3: Replace “$(u_0, u_1, u_2, u_3) = (0, 1, 0, 0)$” with “$(u_0, u_1, u_2) = (0, 1, 0)$”

Page 137, line 14: Replace “both these types” with “both types”

Page 138, line -12: Replace “if $N \neq M$ is an ideal in $R$ with $M \subseteq N \subset R$,” with “if $N \not\subset M$ is an ideal in $R$,”

Page 144, line 3: Replace “$\sum_{n \geq 0} f_n(x)$” with “$\sum_{m \geq 0} f_m(x)$”

Page 143, line 4: Replace “$f_n(x) = \sum_{\alpha \in \mathbb{Z}_2^N \atop |\alpha| = m} c_\alpha x^\alpha$” with “$f_m(x) = \sum_{\alpha \in \mathbb{Z}_2^N \atop |\alpha| = m} c_\alpha x^\alpha$”

Page 143, line 5: Replace the display with $h_m = f_m g_0 + f_{m-1} g_1 + \cdots + f_0 g_m$.”

Page 143, line 7: Replace “$n$” with “$m$”

Page 147, part a of Exercise 2: Replace “$V(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$” with “$V(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$”

Page 147, part b of Exercise 2: Replace “$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^4 \rangle$” with “$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^4 \rangle$”

Page 149, line -18: Replace “note that that” with “note that the”

Page 152, line 6: Replace “Proposition (5.9)” with “Proposition (5.15)”

Page 152, line 3 of Exercise 3: Replace “$f_2 = 6y - x^3 + 9x$,” with “$f_2 = 6y - x^3 + 9x = 0,$”

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Page 152, line 1 of part e of Exercise 3: Replace “Res\((f_1, f_2, y)\)” with “Res\(^y\)(f_1, f_2) as defined in (5.14) of Chapter 3”

Page 152, line 4 of part e of Exercise 3: Replace “Res\((f_1, f_2, x)\)” with “Res\(^x\)(f_1, f_2)”

Page 156, part c of Exercise 10, line 4: Replace “\(A_i \Leftrightarrow f(p_i) = \lambda\)” with “\(A_i \Leftrightarrow f(p_i) = \lambda\)”

Page 160, line –2: Replace “\(\ker(M) \cap \mathbb{Z}_{\geq 0}\)” with “\(\ker(M) \cap \mathbb{Z}\)”

Page 161, line –3: Replace “\(S = \{1 + g : \text{LT}(g) < 1\}\)” with “\(S = \{1 + g : g = 0 \text{ or } \text{LT}(g) < 1\}\)”

Page 163, line 1 of part a of Exercise 5: Replace “let \(h \in A\)” with “let \(h \in \text{Loc}_>(A)\)”

Page 163, line 1 of part b of Exercise 5: Replace “Let \(r \in R\)” with “Let \(r \in \text{Loc}_>(A)\)”

Page 166, lines –20 and –18: Replace “\(t^\alpha > t'^\alpha x^\beta\)” with “\(t^\alpha > t'^\alpha x^\beta\)” (twice)

Page 171, line 3 of part c of Exercise 8: Replace “\(1/(1 + h)\)” with “\(1/(1 + g)\)”

Page 172, line 2 of Exercise 11: Replace “(for local orders)” with “(for degree-anticompatible orders)”

Page 172, line 1 of part a of Exercise 11: Replace “Let > be a local order” with “Let > be a degree-anticompatible order”

Page 193, line –2: Replace “When \(M\) and \(N\) are free modules,” with “When \(M = R^l\) and \(N = R^m\),”

Page 197, line –10: Besides the 1994 paper [PW] by Park and Woodburn, we should also mention two other papers that deal with algorithmic aspects of the Quillen-Suslin result:


Page 199, line 4: “Equivalently, we think” should be “Equivalently, we think”

Page 200, line 18: Replace “one-one” with “one-to-one”

Page 201, line –2 of proof of Proposition (1.11): Replace “\(\sum c_i m_i\)” with “\(\sum c_i f_i\)”

Page 203, line 1: Replace “Let \(\varphi : M \to N\)” with “Let \(\varphi : M \to N\) be an \(R\)-module homomorphism.”

Page 203, line 2 of part a of Exercise 23: Replace “\(\{af : a \in I, f \in M\}\)” with “\(\{\sum_{i=1}^\ell a_i f_i : a_i \in I, f_i \in M\text{ for } i = 1, \ldots, \ell\}\)”.

Page 203, line –16: Replace “We let \(R = k[x, y]\)” with “Let \(R = k[x, y]\), where \(k\) is a field of characteristic different from 2,”

Page 203, lines –9 to –1: Delete and replace with the following:

a. Verify that \(f = (f_1, f_2, f_3)^T = (1, -x/2, -1/2)^T \in R^3\) satisfies \((1 + x)f_1 + (1 - y)f_2 + (x + y)f_3 = 1\).
b. Let $I$ be the $3 \times 3$ identity matrix. Verify that the columns $g_1, g_2, g_3$ of the matrix $I - f \cdot A$ span $\ker A$. Hint: If $A\tilde{f} = 0$, then $\tilde{f} = (I - f \cdot A)\tilde{f}$ is a linear combination of the columns of $I - f \cdot A$.

c. Show that $\{g_1, g_2\}$ is a basis of $\ker A$. (Unfortunately, the result of part c is special to the choice of $f$ made in part a. If $f$ is an arbitrary solution of $A\tilde{f} = 1$, then the first two columns of $I - f \cdot A$ need not give a basis of the kernel.)

Page 204, part b of Exercise 27, line 2: Replace “of $f$ is a nonzero element of $R$” with “of $\tilde{f}$ is a nonzero element of $k$”

Page 205, line 5: Replace “show that $M$” with “then $M$”

Page 206, line 6: Replace “$R^t$ to $R^m$” with “$R^m$ to $R^t$”

Page 206, line 1: Replace “(compare Exercise 6 and the discussion preceding Exercise 7)” with “(compare Exercise 11 and the discussion preceding Proposition (1.10))”?

Page 206, line 3, Replace “column $e_2$” with “column $e_1$”

Page 206, line 5, Replace “row 2 column 1” with “row 1 column 2”

Page 210, lines −9 and −8: Replace “(see Exercise 5 below)” with “(see Exercise 11 of §3)”

Page 211, Exercise 1: Replace “Show” with “Assuming conditions a and b, show”

Page 215, line 10: Replace “$M_{M+\ell}$” with “$M_{N+\ell}$”

Page 215, bottom line: Replace“(1.6)” with “(1.5)”

Page 219, part a of Exercise 2, line 2: Replace “$(dp, C)$” with “$(dp, c)$”

Page 219, line −5: Replace “letter $c$” with “letter $C$”

Page 219, line −3: Replace “lower-case $c$” with “upper-case $C$”

Page 219, lines −2 and −1: Replace “$(dp, C)$” with “$(dp, c)$”

Page 223, line −5: Replace “$\sum_{k=1}^{s} a_{ijk} g_k$” with “$\sum_{\ell=1}^{s} a_{ij\ell} g_\ell$”

Page 223, line −4: Replace “$a_{ijk} \in R$, and $\mathbf{lt}(a_{ijk} g_k) \leq \mathbf{lt}(S(g_i, g_j))$ for all $i, j, k$” with “$a_{ij\ell} \in R$, and $\mathbf{lt}(a_{ij\ell} g_\ell) \leq \mathbf{lt}(S(g_i, g_j))$ for all $i, j, \ell$”

Page 224, line 12: Replace “Exercise 1” with “Exercise 2”

Page 227, line −2: Replace “$(AG I_t - AB)$” with “$(AD I_t - AB)$”.

Page 229, line 16: Replace “the $t$ vectors” with “the $s$ vectors”

Page 229, line 18: Replace “$1 \leq k \leq t$” with “$1 \leq k \leq s$”

Page 231, line 2: Replace “$R^{m+t+s}$” with “$R^{m+t+s}$”

Page 232, line 4 of Exercise 9: Replace “$((a_1, \ldots, a_s) \in R^s$ such that $a_1, \ldots, a_s$” with “$(a_1, \ldots, a_s) \in R^s$ such that $a_1, \ldots, a_s$”

Page 237, line −6: Replace “Hence” with “If $s > 1$, then”
Page 237, line -2: Add the sentence “If $s = 1$, then $(1 - a_1)f_1 = 0$. This implies $f_1 = 0$, which contradicts $M \neq 0$."

Page 239, line 1: Replace “matrix of $M/\mathfrak{m}M.$” with “matrix of $M/\mathfrak{m}M$?"

Page 239, line 10: Replace “columns of $M$” with “columns of $A$”

Page 239, line 13: Replace “in $P/IP$” with “in $M/IM$”

Page 240, line 4: Replace “have have” with “have”

Page 242, line 14: Replace “$m \times 1$ matrix” with “$r \times 1$ matrix”

Page 242, Proposition (4.11): Replace “$Q$ be a local ring, $M$ a finitely generated $Q$-” with “$R$ be a local ring, $M$ a finitely generated $R$-”

Page 243, line -16: Replace “$M/\mathfrak{m}M$ Since” with “$M/\mathfrak{m}M$. Since”

Page 245, part c of Exercise 10: Replace “$0 = F_0(M) \subset F_1(M) \subset \cdots \subset F_{s+1} = R$” with “$0 = F_{-1}(M) \subset F_0(M) \subset \cdots \subset F_s = R$”

Page 248, line -9: Replace “Exercise 12” with “Exercise 28”

Page 253, line -6: Replace with “$M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle$”

Page 254, line 1: Replace with “$M = \text{ideal}(y*z-x*w,y^3-x^2*z,x*z^2-y^2*w,z^3-y*w^2)$”

Page 260, line after second display: Replace “im($\varphi_2$) = Syz($G_1$)” with “im($\varphi_2$) = Syz($G_0$) = ker($\varphi_1$) in $F_1$”

Page 260, lines 1 and 2 after second display: Replace “obtain $\varphi_i : F_i \rightarrow F_{i-1}$, where im($\varphi_i$) = Syz($G_{i-1}$) and $G_i \subset R^n$ is a Gröbner” with “obtain $\varphi_{i+1} : F_{i+1} \rightarrow F_i$, where im($\varphi_{i+1}$) = Syz($G_{i-1}$) = ker($\varphi_i$) in $F_i$ and $G_i \subset F_i = R^n$ is a reduced Gröbner”

Page 260, lines 2 and 3 above display (2.5): Replace “the leading terms of the reduced Gröbner basis $G_\ell$” with “the reduced Gröbner basis $G_\ell$ of Syz($G_{\ell-1}$) $\subset F_\ell$ is either empty or its leading terms”

Page 260, display (2.5): Add $\varphi_{\ell-1}$ above the second arrow and put a period at the end of the display.

Page 260, line after display (2.5): Replace “and the leading” with “When $G_\ell = \emptyset$, ker($\varphi_\ell$) = $\{0\}$ and $\varphi_\ell$ is injective. Thus we can extend (2.5) to a free resolution of length $\ell \leq n$ by adding a zero at the left. Otherwise, the leading”

Page 260, three lines below display (2.5): Replace “Syz($G_{\ell-1}$) is a free module” with “$R^n/k\text{er}($\varphi_\ell$) \cong \text{im}($\varphi_\ell$) = ker($\varphi_{\ell-1}$) is a free module”

Page 260, four lines below display (2.5): Replace “we can extend (2.5)” should be “we can replace $F_\ell$ with the free module ker($\varphi_{\ell-1}$) and extend (2.5)”

Page 263, line 1: Replace “from (1.8)” with “from (1.7)”

Page 263, line 6: Replace “see (1.16)” with “(see (1.14))”

Page 264, Exercise 8: Add the following new part d:
d. Show that \( R^t/M \) is also a free module. Hint: Let \( N \subset R^t \) be the free submodule generated by the standard basis vectors that are not leading terms of elements of \( G \). Use the division algorithm with respect to \( G \) to show that the induced map \( N \to R^t/M \) is an isomorphism.

Page 265, part a of Exercise 11: Replace “of the the” with “of the”

Page 265, part b of Exercise 11, line 2: Replace \((-1) \det(A_i)\), where \( A_i \)” with “\((-1) \det(\mathbf{A}_i)\), where \( \mathbf{A}_i \)”

Page 265, part d of Exercise 11, line -1: Replace “= pB for some \( B \in R^m \)” with “= pC for some \( C \in R^m \)”

Page 267, line 1 of (3.3) Proposition: Replace “be submodule” with “be a submodule”

Page 269, line 1 of Exercise 3: Replace “finitely generated” with “finitely generated graded”

Page 270, line 6: Replace with “\( M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle \)”

Page 270, line 9: Replace with “\( R(-2) \oplus R(-3)^3 \to R \)”

Page 275, line 2: Replace “\( F_{\ell+2} \xrightarrow{\varphi_{\ell+1}} F_{\ell+1} \)” with “\( F_{\ell+2} \xrightarrow{\varphi_{\ell+2}} F_{\ell+1} \)”

Page 275, line 7: Replace “\(+ c_2\varphi_{\ell-1}(u_m)\)” with “\(+ c_\ell\varphi_{\ell-1}(u_\ell)\)”

Page 275, line 9: Replace “\( i = 2, \ldots, m \)” with “\( i = 2, \ldots, t \)”

Page 279, line 2 of Exercise 14: Replace “\( \psi : G_\ell \to G_{\ell-1} \)” with “\( \psi : G_\ell \to G_{\ell-1} \)”

Page 279, lines 6-7 of Exercise 14: Replace “\( A_{01} = (c_2, \ldots, c_\ell) \)” as in (3.16), and \( A_{10} = (d_2, \ldots, d_m)^T \)” with “\( A_{01} = (c_2, \ldots, c_\ell)^T \)” as in (3.16), and \( A_{01} = (d_2, \ldots, d_m)^T \)”

Page 279, line 10 of Exercise 14: Replace “\( B_\ell = A_{00} - A_{01}A_{11}^{-1}A_{10} \)” with “\( B_\ell = A_{11} - A_{10}A_{00}^{-1}A_{01} \)”

Page 279, line 11 of Exercise 14: Replace “What’s remarkable is that this formula is identical to” with “This formula is a slight variation of the formula in”

Page 289, line 3 of Definition (4.16): “to the minimal” should be “to be the minimal”

Page 290, line 12: Replace “for \( S/J \) to” with “for \( R/J \) to”

Page 293, line 3: Replace “\( \tilde{c} = p_1q_2 - p_1q_2 \)” with “\( \tilde{c} = p_1q_2 - p_2q_1 \)”

Page 293, line -19: Replace “\( \text{GCD}(a_1, \ldots, a_m) = 1 \)” with “\( \text{GCD}(a_1, \ldots, a_m, c) = 1 \)”

Page 297, part a of Exercise 12, line 3: Replace “\( R^G \)” with “\( S^G \)”

Page 297, line -3: Replace “\( R^G \)” with “\( S^G \)”

Page 303, part d of Exercise 25: Replace “parts b, c and d” with “parts b and d”

Page 308, line before Exercise 4: Add a new sentence “We also regard \( Q \) as a face of itself.”

Page 308, line following Exercise 4: Replace “Every face” with “Every proper face”

Page 314, line 3 of Exercise 1: Replace “You already did a special case of this in Exercise 2 of Chapter 3, §2” with “This is a special case of Exercise 2 of Chapter 3, §2”
Page 314, the last row of the matrix in display (2.4): Replace “$c_0 - x$” with “$c_0 - z$”
Page 314, part a of Exercise 2, line 4: Replace “$st^2$” with “$s^2t$”
Page 319, line 1 of Exercise 6: Replace “Then” with “Use the bracket notation introduced in Theorem (3.5) of Chapter 3, §3 to”
Page 325, display (3.9): Replace “$F(x_1, \ldots, x_n)$” with “$F(x_1, \ldots, x_N)$”
Page 325, line 1 of proof of Lemma (3.10): Replace “$m = \sum_{i=1}^{n} a_i e_i$” with “$m = \sum_{i=1}^{n} b_i e_i$”
Page 325, line 2 of Exercise 4: Replace “Exercise 3” with “Exercise 7 of §1”
Page 327, line 9: In the statement of Theorem (3.13), replace “$A = \{m_1, \ldots, m_l\} \subset \mathbb{Z}^{n \geq 0}$” with “$A = \{m_1, \ldots, m_l\} \subset \mathbb{Z}^n$”
Page 328, line 11: In two places, replace “$x_0, \ldots, x_N$” with “$x_1, \ldots, x_N$”
Page 331, part d of Exercise 11, line 2: Replace “$x_1, \ldots, x_n$” with “$x_1, \ldots, x_N$”
Page 331, part d of Exercise 11, line 3: Replace “$x_1, \ldots, x_n$” with “$x_1, \ldots, x_N$”
Page 334, line -11: Replace “which is the called” with “which is called”
Page 334, line -10: Replace “If $S$ is subset of” with “If $S$ is a subset of”
Page 339, line 12: Replace “part b” with “part c”
Page 339, line 18: Replace “Exercise 5” with “Exercise 6”
Page 342, line 1: Replace “$\mu \cdot a_Q(\nu) \geq 0$” with “$\mu \cdot a_Q(\nu)/\|\nu\| \geq 0$”
Page 343, 7 lines below display (5.2): Replace “equivalent” with “equivalent to”
Page 352, part e of Exercise 4, line 1: Replace “$d \mapsto d/t$” with “$d \mapsto d/t^8$”
Page 359, line -7: Replace “polyedral” with “polyhedral”
Page 359, line 4 of Definition (6.4): Replace “is a face” with “is either empty or a face”
Page 365, Figure 7.9: The figure is wrong. Here is the correct figure.

Page 373, line 3: Replace “Chapter 2” with “Chapter 3”
Page 416, part b of Exercise 4, line 6: Replace “\(g_3 = (2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)\)” with “\(g_3 = (2xy^2 + y^3, x^2y + 2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)\)”

Page 417, part c of Exercise 5, last line: Replace “if \(k \geq 3\)” with “if \(k \geq 4\)”

Page 422, part d of Exercise 8: Replace “\(M(\Delta', r)\)” with “\(M(\Delta, r)\)”

Page 423, line 7: Replace “expression (3.19)” with “expression (3.18)”

Page 425, Exercise 14, line 2: Replace “hereditary complex” with “hereditary simplicial complex”

Page 431, line −6: The left-hand side of the equation should be “\(\{x^2 - y, yz + xz - y^2\}\)”

Page 433, line 1: Replace “that \(w\)” with “that \(w\)”

Page 438, line −18: Replace “is the positive orthant” with “in the positive orthant”

Page 440, third display: Replace “\(\langle \text{in}_{\text{new}}(G_{\text{old}}) \rangle\)” with “\(\langle \text{LT}_{\text{new}}(\langle \text{in}_{\text{new}}(G_{\text{old}}) \rangle) \rangle\)”

Page 440, line −9: In two places, replace “\(q_{j,g}\)” with “\(p_{j,g}\)”

Page 444, line 10: “\(w_t \cdot v_1 = 6\)” should be “\(w_t \cdot v_1 = 11\)”

Page 444, line 14: “\(v_2 = (0, -, -1)\)” should be “\(v_3 = (0, 1, -1)\)”

Page 473, line −3: Replace “\(\langle x_1^{n_1 - 1} - 1, \ldots, x_m^{n_m - 1} - 1 \rangle\)” with “\(\langle x_1^{n_1} - 1, \ldots, x_m^{n_m} - 1 \rangle\)”

Page 474, line 6: Replace “\(\langle x_1^{n_1} - 1, \ldots, x_m^{n_m} - 1 \rangle\)” with “\(\langle x_1^{n_1 - 1} - 1, \ldots, x_m^{n_m - 1} \rangle\)”

Page 496, line 7: Replace “of elements” with “of nonzero elements”

Page 496, line −3: Replace “are verified” with “are satisfied”

Page 502, line 10: Replace “\(x_1^4x_2^4\)” with “\(x_1^3x_2^3\)”

Page 553, first column, line −14: Replace “Faugère, C.” with “Faugère, J.-C.”