

## Using Algebraic Geometry, *second edition*

December 29, 2024

### Errata:

Page 9, line 4: Replace “ $\alpha \cdot \mathbf{w}_1$  and  $\alpha \cdot \mathbf{w}_1$ ” with “ $\alpha \cdot \mathbf{w}_1$  and  $\beta \cdot \mathbf{w}_1$ ”

Page 18, line 6: “second polynomial” should be “first polynomial”

Page 28, line 4: Replace “last” with “first”

Page 39, line –7: Replace “ $\mathbf{V}(I) \subset \mathbb{C}^n$ ” with “ $\mathbf{V}(I) = \{a \in \mathbb{C}^n : f(a) = 0 \text{ for all } f \in I\}$ ”

Page 45, part c of Exercise 9: Delete and replace with “Show that  $1 = \sum_j (1/p_j(a_j))p_j$ .”

Page 46, Exercise 11: Add a new part c of the exercise as follows:

- c. For readers familiar with the Hermitian inner product  $\langle z, w \rangle = \sum_{i=1}^n z_i \overline{w}_i$  for  $z, w \in \mathbb{C}^n$ , let

$$h_i(x) = \prod_{j \neq i} \langle x - p_i, p_i - p_j \rangle.$$

Show that  $g_i(x) = h_i(x)/h_i(p_i)$  satisfies part b.

Page 54, line 5: Replace “ $L(x - \sum_j c_j x^{\alpha(j)}) = 0$ ” with “ $L(x^\alpha - \sum_j c_j x^{\alpha(j)}) = 0$ ”

Page 55, line 2 of Exercise 2: Replace “ $x^\alpha > x_1^a$ ” with “ $x^\alpha \geq x_1^a$ ”

Page 55, line 2 of Exercise 3: Replace “let  $x^\alpha$  be” with “let  $x^\alpha = x_1^{a_1} \cdots x_n^{a_n}$  be”

Page 65, line 7 after the second display: Replace “ $i_1 > \cdots > i_l$ ” with “ $i_1 < \cdots < i_l$ ”

Page 71, lines 4–8 of the proof of Theorem (5.2): Replace “Hence we will only ... invertible matrix” with “Hence we will only discuss the broad outline of the proof. In the case when  $I$  is radical, it is possible to turn the sketch that follows into a rigorous proof.”

Page 75, line –3: Replace “ $-\text{rem}(p_{i-1}(t), p_{i-2}(t), t)$ ” with “ $-\text{rem}(p_{i-2}(t), p_{i-1}(t), t)$ ”

Page 75, line –2: Replace “division of  $p_{i-1}$  by  $p_{i-2}$ ” with “division of  $p_{i-2}$  by  $p_{i-1}$ ”

Page 81, line 1 of Proposition (1.7): Replace “ $\mathbb{Z}[a_0, \dots, a_l, b_0, \dots, b_m]$ ” with “ $\mathbb{Z}[u_0, \dots, u_l, v_0, \dots, v_m]$ ”

Page 82, line 2 of Exercise 7: Replace “ $\deg(r) < \deg(g)$ ” with “ $\deg(r) < \deg(f)$ ”

Page 92, line 8: Replace “degrees  $d_1, \dots, d_n$ ” with “ $d_0, \dots, d_n$ ”

Page 97, lines 21 and line 24: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 100, line –2: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 101, part b of Exercise 10, line 2: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 102, part c of Exercise 11, line 2: Replace “multiplication by  $(-1)^n$ ” with “multiplication by  $(-1)^{n-1}$ ”

Page 102, part d of Exercise 11: Replace “Theorem (3.5)” with “Theorem (3.4)”

Page 106, line 2 of Exercise 8: Replace “total degree 420” with “total degree 210”

Page 107, line 2 of the proof of Proposition (4.7): Replace “ $(n - 1)!$  ways” with “ $n!$  ways”

Page 108, line 2 of Exercise 11: Replace “ $D'_3$ ” with “ $D'_2$ ”

Page 109, line 3: Replace “Exercise 10” with “Exercise 11”

Page 113, part d of Exercise 22: Replace part d with “Use part c to show that the determinant in (2.8) vanishes whenever  $F_0 = F_1 = F_2 = 0$  has a nontrivial solution.”

Page 120, line 6 after display (5.12): Replace “ $u_1 = \cdots = u_n = 0$ ” with “ $u_1 = \cdots = u_{n-1} = 0$ ”

Page 123, line 11: Replace “ $A = \mathbb{C}(u)[x_1, \dots, x_n]/\langle u - x_n, f_1, \dots, f_n \rangle$ ” with “ $A = \mathbb{C}(u_0)[x_1, \dots, x_n]/\langle u - x_n, f_1, \dots, f_{n-1} \rangle$ ”

Page 123, line 19: Replace “ $\hat{A} = \mathbb{C}(u_0)[x_1, \dots, x_{n-1}]/\langle \hat{f}_1, \dots, \hat{f}_n \rangle$ ” with “ $\hat{A} = \mathbb{C}(u_0)[x_1, \dots, x_{n-1}]/\langle \hat{f}_1, \dots, \hat{f}_{n-1} \rangle$ ”

Page 125, last display: Replace “ $F_0 = \cdots = F_n = 0$ ” with “ $F_1 = \cdots = F_n = 0$ ”

Page 126, line 8: Replace “ $f_0 = \cdots = f_n = 0$ ” with “ $f_1 = \cdots = f_n = 0$ ”

Page 129, line -2: Replace “ $x^\alpha/x_i^{d_i}$  has degree  $\leq d - d_i$ ” with “ $x^\beta/x_i^{d_i}$  has degree  $\leq d - d_i$ ”

Page 131, lines 1 and 2 following second-to-last display: Replace “Exercise 12 of Chapter 2, §4” with “Exercise 12 of Chapter 2, §2”

Page 134, line 2 of Exercise 3: Replace “ $(u_0, u_1, u_2, u_3) = (0, 1, 0, 0)$ ” with “ $(u_0, u_1, u_2) = (0, 1, 0)$ ”

Page 137, line 14: Replace “both these types” with “both types”

Page 138, line -12: Replace “if  $N \neq M$  is an ideal in  $R$  with  $M \subset N \subset R$ ,” with “if  $N \not\subset M$  is an ideal in  $R$ ,”

Page 138, lines -10 to -8: Replace “Therefore  $M \dots$  contained in  $M$ .” with “Therefore, every proper ideal of  $R$  is contained in  $M$ . Hence  $M$  is maximal and is the only maximal ideal of  $R$ .”

Page 142, line 3: Replace “ $\sum_{n \geq 0} f_n(x)$ ” with “ $\sum_{m \geq 0} f_m(x)$ ”

Page 143, line 4: Replace “ $f_n(x) = \sum_{\substack{\alpha \in \mathbb{Z}_{\geq 0}^n \\ |\alpha| = n}} c_\alpha x^\alpha$ ” with “ $f_m(x) = \sum_{\substack{\alpha \in \mathbb{Z}_{\geq 0}^n \\ |\alpha| = m}} c_\alpha x^\alpha$ ”

Page 143, line 5: Replace the display with  $h_m = f_m g_0 + f_{m-1} g_1 + \cdots + f_0 g_m$ .

Page 143, line 7: Replace “ $n$ ” with “ $m$ ”

Page 147, part c of Exercise 1: Delete “uniquely”

Page 147, part a of Exercise 2: Replace “ $\mathbf{V}(x^2 - 2x + y^2, x^2 - 4x + 4y^4)$ ” with “ $\mathbf{V}(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$ ”

Page 147, part b of Exercise 2: Replace “ $\langle x^2 - 2x + y^2, x^2 - 4x + 4y^4 \rangle$ ” with “ $\langle x^2 - 2x + y^2, x^2 - 4x + 4y^2 \rangle$ ”

Page 149, line -18: Replace “note that that” with “note that the”

Page 152, line 6: Replace “Proposition (5.9)” with “Proposition (5.15)”

Page 152, line 3 of Exercise 3: Replace “ $f_2 = 6y - x^3 + 9x$ ,” with “ $f_2 = 6y - x^3 + 9x = 0$ ,”

Page 152, line 1 of part e of Exercise 3: Replace “ $\text{Res}(f_1, f_2, y)$ ” with “ $\text{Res}^y(f_1, f_2)$  as defined in (5.14) of Chapter 3”

Page 152, line 4 of part e of Exercise 3: Replace “ $\text{Res}(f_1, f_2, x)$ ” with “ $\text{Res}^x(f_1, f_2)$ ”

Page 156, part c of Exercise 10, line 4: Replace “ $A_i \Leftrightarrow f(p) = \lambda$ ” with “ $A_i \Leftrightarrow f(p_i) = \lambda$ ”

Page 160, line –2: Replace “ $\ker(M) \cap \mathbb{Z}_{\geq 0}^n$ ” with “ $\ker(M) \cap \mathbb{Z}^n$ ”

Page 161, line –3: Replace “ $S = \{1 + g : \text{LT}(g) < 1\}$ ” with “ $S = \{1 + g : g = 0 \text{ or } \text{LT}(g) < 1\}$ ”

Page 163, line 1 of part a of Exercise 5: Replace “let  $h \in A$ ” with “let  $h \in \text{Loc}_{>}(A)$ ”

Page 163, line 1 of part b of Exercise 5: Replace “Let  $r \in R$ ” with “Let  $r \in \text{Loc}_{>}(A)$ ”

Page 166, lines –20 and –18: Replace “ $t^a > t^{a'} x^\beta$ ” with “ $t^a >' t^{a'} x^\beta$ ” (twice)

Page 171, line 3 of part c of Exercise 8: Replace “ $1/(1 + h)$ ” with “ $1/(1 + g)$ ”

Page 172, line 2 of Exercise 11: Replace “(for local orders)” with “(for degree-anticompatible orders)”

Page 172, line 1 of part a of Exercise 11: Replace “Let  $>$  be a local order” with “Let  $>$  be a degree-anticompatible order”

Page 193, line –2: Replace “When  $M$  and  $N$  are free modules,” with “When  $M = R^l$  and  $N = R^m$ ,”

Page 197, line –10: Besides the 1994 paper [PW] by Park and Woodburn, we should also mention two other papers that deal with algorithmic aspects of the Quillen-Suslin result:

L. Caniglia, G. Cortiñas, S. Danón, J. Heintz, T. Krick and P. Solernó (working group N. Fitchas), *Algorithmic aspects of Suslin’s proof of Serre’s conjecture*, Comput. Complexity **3** (1993), 31–55

N. Fitchas and A. Galligo, *Nullstellensatz effectif et conjecture de Serre (théorème de Quillen-Suslin) pour le calcul formel*, Math. Nachr. **149** (1990), 231–253.

Page 199, line 4: “Equivalently. we think” should be “Equivalently, we think”

Page 200, line 18: Replace “one-one” with “one-to-one”

Page 201, line –2 of proof of Proposition (1.11): Replace “ $\sum c_i m_i$ ” with “ $\sum c_i f_i$ ”

Page 203, line 1: Replace “Let  $\varphi : M \rightarrow N$ .” with “Let  $\varphi : M \rightarrow N$  be an  $R$ -module homomorphism.”

Page 203, line 2 of part a of Exercise 23: Replace “ $\{af : a \in I, f \in M\}$ ” with “ $\{\sum_{i=1}^{\ell} a_i f_i : a_i \in I, f_i \in M \text{ for } i = 1, \dots, \ell\}$ ”.

Page 203, line –16: Replace “We let  $R = k[x, y]$ ” with “Let  $R = k[x, y]$ , where  $k$  is a field of characteristic different from 2,”

Page 203, lines –9 to –1: Delete and replace with the following:

- a. Verify that  $\mathbf{f} = (f_1, f_2, f_3)^T = (1, -x/2, -1/2)^T \in R^3$  satisfies  $(1+x)f_1 + (1-y)f_2 + (x+xy)f_3 = 1$ .
- b. Let  $I$  be the  $3 \times 3$  identity matrix. Verify that the columns  $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$  of the matrix  $I - \mathbf{f} \cdot A$  span  $\ker A$ . Hint: If  $A\tilde{\mathbf{f}} = 0$ , then  $\tilde{\mathbf{f}} = (I - \mathbf{f} \cdot A)\mathbf{f}$  is a linear combination of the columns of  $I - \mathbf{f} \cdot A$ .
- c. Show that  $\{\mathbf{g}_1, \mathbf{g}_2\}$  is a basis of  $\ker A$ . (Unfortunately, the result of part c is special to the choice of  $\mathbf{f}$  made in part a. If  $\mathbf{f}$  is an arbitrary solution of  $A\mathbf{f} = 1$ , then the first two columns of  $I - \mathbf{f} \cdot A$  need not give a basis of the kernel.)

Page 204, part b of Exercise 27, line 2: Replace “of  $f$  is a nonzero element of  $R$ ” with “of  $\mathbf{f}$  is a nonzero element of  $k$ ”

Page 205, line 5: Replace “show that  $M$ ” with “then  $M$ ”

Page 205, line 6: Replace “ $R^l$  to  $R^m$ ” with “ $R^m$  to  $R^l$ ”

Page 206, line 1: Replace “(compare Exercise 6 and the discussion preceding Exercise 7)” with “(compare Exercise 11 and the discussion preceding Proposition (1.10))?”

Page 206, line 3, Replace “column  $e_2$ ” with “column  $e_1$ ”

Page 206, line 5, Replace “row 2 column 1” with “row 1 column 2”

Page 209, line 2 of part c of Proposition (2.3): Replace “ $\epsilon_1, \dots, \epsilon_t$ ” with “ $\mathbf{e}_1, \dots, \mathbf{e}_t$ ”

Page 209, line 4 of part c of Proposition (2.3): Replace “ $(\mathbf{m}_{ij}/\mathbf{m}_i)\epsilon_i - (\mathbf{m}_{ij}/\mathbf{m}_j)\epsilon_j$ ” with “ $(\mathbf{m}_{ij}/\mathbf{m}_i)\mathbf{e}_i - (\mathbf{m}_{ij}/\mathbf{m}_j)\mathbf{e}_j$ ”

Page 210, second display: Replace “ $m_s = x^{\alpha_s}\mathbf{e}_i$ ” with “ $\mathbf{m}_s = x^{\alpha_s}\mathbf{e}_i$ ”

Page 210, lines –9 and –8: Replace “(see Exercise 5 below)” with “(see Exercise 11 of §3)”

Page 211, Exercise 1: Replace “Show” with “Assuming conditions a and b, show”

Page 215, line 10: Replace “ $M_{M+\ell}$ ” with “ $M_{N+\ell}$ ”

Page 215, bottom line: Replace “(1.6)” with “(1.5)”

Page 219, part a of Exercise 2, line 2: Replace “ $(\mathbf{dp}, \mathbf{C})$ ” with “ $(\mathbf{dp}, \mathbf{c})$ ”

Page 219, line –5: Replace “letter  $\mathbf{C}$ ” with “letter  $\mathbf{c}$ ”

Page 219, line –3: Replace “lower-case  $\mathbf{c}$ ” with “upper-case  $\mathbf{C}$ ”

Page 219, lines –2 and –1: Replace “ $(\mathbf{dp}, \mathbf{C})$ ” with “ $(\mathbf{dp}, \mathbf{c})$ ”

Page 223, line –5: Replace “ $\sum_{k=1}^s a_{ijk}\mathbf{g}_k$ ” with “ $\sum_{\ell=1}^s a_{ij\ell}\mathbf{g}_\ell$ ”

Page 223, line –4: Replace “ $a_{ijk} \in R$ , and  $\text{LT}(a_{ijk}\mathbf{g}_k) \leq \text{LT}(S(\mathbf{g}_i, \mathbf{g}_j))$  for all  $i, j, k$ ” with “ $a_{ij\ell} \in R$ , and  $\text{LT}(a_{ij\ell}\mathbf{g}_\ell) \leq \text{LT}(S(\mathbf{g}_i, \mathbf{g}_j))$  for all  $i, j, \ell$ ”

Page 224, line 12: Replace “Exercise 1” with “Exercise 2”

Page 227, line –2: Replace “ $(AG \ I_t - AB)$ ” with “ $(AD \ I_t - AB)$ ”.

Page 229, line 16: Replace “the  $t$  vectors” with “the  $s$  vectors”

Page 229, line 18: Replace “ $1 \leq k \leq t$ ” with “ $1 \leq k \leq s$ ”

Page 231, line 2: Replace “ $R^{n+t+s}$ ” with “ $R^{m+t+s}$ ”

Page 232, line 4 of Exercise 9: Replace “ $(\mathbf{a}_1, \dots, \mathbf{a}_s) \in R^s$  such that  $\mathbf{a}_1, \dots, \mathbf{a}_s$ ” with “ $(a_1, \dots, a_s) \in R^s$  such that  $a_1, \dots, a_s$ ”

Page 237, line –6: Replace “Hence” with “If  $s > 1$ , then”

Page 237, line –2: Add the sentence “If  $s = 1$ , then  $(1 - a_1)f_1 = 0$ . This implies  $f_1 = 0$ , which contradicts  $M \neq 0$ .”

Page 239, line 1: Replace “matrix of  $M/\mathfrak{m}M$ .” with “matrix of  $M/\mathfrak{m}M$ ?”

Page 239, line 10: Replace “columns of  $M$ ” with “columns of  $A$ ”

Page 239, line 13: Replace “in  $P/IP$ ” with “in  $M/IM$ ”

Page 240, line 4: Replace “have have” with “have”

Page 242, line 14: Replace “ $m \times 1$  matrix” with “ $r \times 1$  matrix”

Page 242, Proposition (4.11): Replace “ $Q$  be a local ring,  $M$  a finitely generated  $Q$ ” with “ $R$  be a local ring,  $M$  a finitely generated  $R$ ”

Page 243, line –16: Replace “ $M/\mathfrak{m}M$  Since” with “ $M/\mathfrak{m}M$ . Since”

Page 245, part c of Exercise 10: Replace “ $0 = F_0(M) \subset F_1(M) \subset \dots \subset F_{s+1} = R$ ” with “ $0 = F_{-1}(M) \subset F_0(M) \subset \dots \subset F_s = R$ ”

Page 248, line –9: Replace “*Exercise 12*” with “*Exercise 28*”

Page 253, line –6: Replace with “ $M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle$ ”

Page 254, line 1: Replace with “ $\mathfrak{M} = \text{ideal}(y^*z - x^*w, y^3 - x^2z, x^*z^2 - y^2w, z^3 - y^*w^2)$ ”

Page 260, line after second display: Replace “ $\text{im}(\varphi_2) = \text{Syz}(G_1)$ ” with “ $\text{im}(\varphi_2) = \text{Syz}(G_0) = \ker(\varphi_1)$  in  $F_1$ ”

Page 260, lines 1 and 2 after second display: Replace “obtain  $\varphi_i : F_i \rightarrow F_{i-1}$ , where  $\text{im}(\varphi_i) = \text{Syz}(G_{i-1})$  and  $\mathcal{G}_i \subset R^{r_i}$  is a Gröbner” with “obtain  $\varphi_{i+1} : F_{i+1} \rightarrow F_i$ , where  $\text{im}(\varphi_{i+1}) = \text{Syz}(G_{i-1}) = \ker(\varphi_i)$  in  $F_i$  and  $\mathcal{G}_i \subset F_i = R^{r_i}$  is a reduced Gröbner”

Page 260, lines 2 and 3 above display (2.5): Replace “the leading terms of the reduced Gröbner basis  $\mathcal{G}_\ell$ ” with “the reduced Gröbner basis  $\mathcal{G}_\ell$  of  $\text{Syz}(G_{\ell-1}) \subset F_\ell$  is either empty or its leading terms”

Page 260, display (2.5): Add  $\varphi_{\ell-1}$  above the second arrow and put a period at the end of the display.

Page 260, line after display (2.5): Replace “and the leading” with “When  $\mathcal{G}_\ell = \emptyset$ ,  $\ker(\varphi_\ell) = \{0\}$  and  $\varphi_\ell$  is injective. Thus we can extend (2.5) to a free resolution of length  $\ell \leq n$  by adding a zero at the left. Otherwise, the leading”

Page 260, three lines below display (2.5): Replace “ $\text{Syz}(G_{\ell-1})$  is a free module” with “ $R^t/\ker(\varphi_\ell) \simeq \text{im}(\varphi_\ell) = \ker(\varphi_{\ell-1})$  is a free module”

Page 260, four lines below display (2.5): Replace “we can extend (2.5)” should be “we can replace  $F_\ell$  with the free module  $\ker(\varphi_{\ell-1})$  and extend (2.5)”

Page 263, line 1: Replace “from (1.8)” with “from (1.7)”

Page 263, line 6: Replace “see (1.16)” with “(see (1.14))”

Page 264, Exercise 8: Add the following new part d:

- d. Show that  $R^t/M$  is also a free module. Hint: Let  $N \subset R^t$  be the free submodule generated by the standard basis vectors that are not leading terms of elements of  $\mathcal{G}$ . Use the division algorithm with respect to  $\mathcal{G}$  to show that the induced map  $N \rightarrow R^t/M$  is an isomorphism.

Page 265, part a of Exercise 11: Replace “of the the” with “of the”

Page 265, part b of Exercise 11, line 2: Replace “ $(-1)\det(A_i)$ , where  $A_i$ ” with “ $(-1)\det(\mathcal{A}_i)$ , where  $\mathcal{A}_i$ ”

Page 265, part d of Exercise 11, line -1: Replace “ $= pB$  for some  $B \in R^m$ ” with “ $= pC$  for some  $C \in R^m$ ”

Page 267, line 1 of **(3.3) Proposition**: Replace “be submodule” with “be a submodule”

Page 269, line 1 of Exercise 3: Replace “finitely generated” with “finitely generated graded”

Page 270, line 6: Replace with “ $M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle$ ”

Page 270, line 9: Replace with “ $R(-2) \oplus R(-3)^3 \rightarrow R$ ”

Page 275, line 2: Replace “ $F_{\ell+2} \xrightarrow{\varphi_{\ell+1}} F_{\ell+1}$ ” with “ $F_{\ell+2} \xrightarrow{\varphi_{\ell+2}} F_{\ell+1}$ ”

Page 275, line 7: Replace “ $+ c_2\varphi_{\ell-1}(u_m)$ ” with “ $+ c_t\varphi_{\ell-1}(u_t)$ ”

Page 275, line 9: Replace “ $i = 2, \dots, m$ ” with “ $i = 2, \dots, t$ ”

Page 279, line 2 of Exercise 14: Replace “ $\psi : G_\ell \rightarrow G_{\ell-1}$ ” with “ $\psi_\ell : G_\ell \rightarrow G_{\ell-1}$ ”

Page 279, lines 6-7 of Exercise 14: Replace “ $A_{01} = (c_2, \dots, c_t)$  as in (3.16), and  $A_{10} = (d_2, \dots, d_m)^T$ ” with “ $A_{10} = (c_2, \dots, c_t)^T$  as in (3.16), and  $A_{01} = (d_2, \dots, d_m)$ ”

Page 279, line 10 of Exercise 14: Replace “ $B_\ell = A_{00} - A_{01}A_{11}^{-1}A_{10}$ ” with “ $B_\ell = A_{11} - A_{10}A_{00}^{-1}A_{01}$ ”

Page 279, line 11 of Exercise 14: Replace “What’s remarkable is that this formula is identical to” with “This formula is a slight variation of the formula in”

Page 289, line 3 of Definition (4.16): “to the minimal” should be “to be the minimal”

Page 290, line 12: Replace “for  $S/J$  to” with “for  $R/J$  to”

Page 293, line 3: Replace “ $\tilde{c} = p_1q_2 - p_1q_2$ ” with “ $\tilde{c} = p_1q_2 - p_2q_1$ ”

Page 293, line -19: Replace “ $\text{GCD}(a_1, \dots, a_m) = 1$ ” with “ $\text{GCD}(a_1, \dots, a_m, c) = 1$ ”

Page 297, part a of Exercise 12, line 3: Replace “ $R^G$ ” with “ $S^G$ ”

Page 297, line -3: Replace “ $R^G$ ” with “ $S^G$ ”

Page 303, part d of Exercise 25: Replace “parts b, c and d” with “parts b and d”

Page 308, line before Exercise 4: Add a new sentence “We also regard  $Q$  as a face of itself.”

Page 308, line following Exercise 4: Replace “Every face” with “Every proper face”

Page 314, line 3 of Exercise 1: Replace “You already did a special case of this in Exercise 2 of Chapter 3, §2” with “This is a special case of Exercise 2 of Chapter 3, §2”

Page 314, the last row of the matrix in display (2.4): Replace “ $c_0 - x$ ” with “ $c_0 - z$ ”

Page 314, part a of Exercise 2, line 4: Replace “ $st^2$ ” with “ $s^2t$ ”

Page 319, line 1 of Exercise 6: Replace “Then” with “Use the bracket notation introduced in Theorem (3.5) of Chapter 3, §3 to”

Page 325, display (3.9): Replace “ $F(x_1, \dots, x_n)$ ” with “ $F(x_1, \dots, x_N)$ ”

Page 325, line 1 of proof of Lemma (3.10): Replace “ $m = \sum_{i=1}^n a_i e_i$ ” with “ $m = \sum_{i=1}^n b_i e_i$ ”

Page 325, line 2 of Exercise 4: Replace “Exercise 3” with “Exercise 7 of §1”

Page 327, line 9: In the statement of Theorem (3.13), replace “ $\mathcal{A} = \{m_1, \dots, m_l\} \subset \mathbb{Z}_{\geq 0}^n$ ” with “ $\mathcal{A} = \{m_1, \dots, m_l\} \subset \mathbb{Z}^n$ ”

Page 328, line 11: In two places, replace “ $x_0, \dots, x_N$ ” with “ $x_1, \dots, x_N$ ”

Page 331, part d of Exercise 11, line 2: Replace “ $x_1, \dots, x_n$ ” with “ $x_1, \dots, x_N$ ”

Page 331, part d of Exercise 11, line 3: Replace “ $x_1, \dots, x_n$ ” with “ $x_1, \dots, x_N$ ”

Page 334, line –11: Replace “which is the called” with “which is called”

Page 334, line –10: Replace “If  $S$  is subset of” with “If  $S$  is a subset of”

Page 339, line 12: Replace “part b” with “part c”

Page 339, line 18: Replace “Exercise 5” with “Exercise 6”

Page 342, line 1: Replace “ $\mu \cdot a_Q(\nu) \geq 0$ ” with “ $\mu \cdot a_Q(\nu) / \|\nu\| \geq 0$ ”

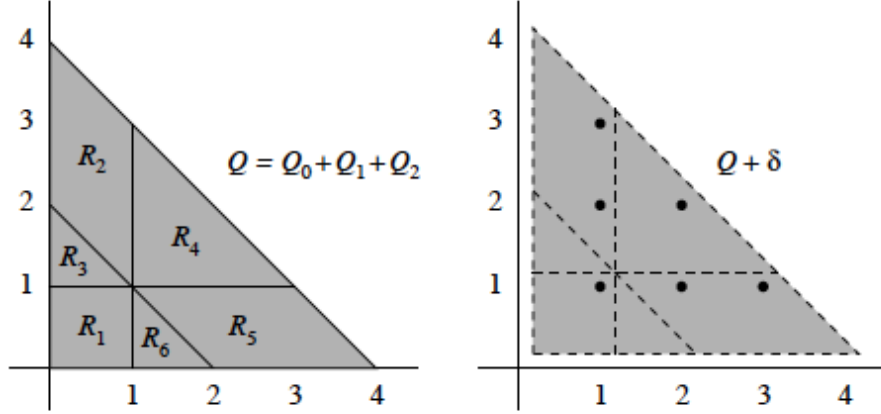
Page 343, 7 lines below display (5.2): Replace “equivalent” with “equivalent to”

Page 352, part e of Exercise 4, line 1: Replace “ $d \mapsto d/t$ ” with “ $d \mapsto d/t^8$ ”

Page 359, line -7: Replace “polyedral” with “polyhedral”

Page 359, line 4 of Definition (6.4): Replace “is a face” with “is either empty or a face”

Page 365, Figure 7.9: The figure is wrong. Here is the correct figure.



Page 373, line 3: Replace “Chapter 2” with “Chapter 3”

Page 416, part b of Exercise 4, line 6: Replace “ $g_3 = (2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)$ ” with “ $g_3 = (2xy^2 + y^3, x^2y + 2xy^2 + y^3, 0, 0, y, -y, 0, -2x - y)$ ”

Page 417, part c of Exercise 5, last line: Replace “if  $k \geq 3$ ” with “if  $k \geq 4$ ”

Page 422, part d of Exercise 8: Replace “ $M(\Delta', r)$ ” with “ $M(\overline{\Delta}, r)$ ”

Page 423, line 7: Replace “expression (3.19)” with “expression (3.18)”

Page 425, Exercise 14, line 2: Replace “hereditary complex” with “hereditary simplicial complex”

Page 431, line -6: The left-hand side of the equation should be “ $\{\underline{x}^2 - y, \underline{yz} + xz - y^2\}$ ”

Page 433, line 1: Replace “that  $w$ ” with “that  $\mathbf{w}$ ”

Page 438, line -18: Replace “is the positive orthant” with “in the positive orthant”

Page 440, third display: Replace “ $\langle \text{in}_{\mathbf{w}_{new}}(G_{old}) \rangle$ ” with “ $\langle \text{LT}_{>_{new}}(\langle \text{in}_{\mathbf{w}_{new}}(G_{old}) \rangle) \rangle$ ”

Page 440, line -9: In two places, replace “ $q_{j,g}$ ” with “ $p_{j,g}$ ”

Page 444, line 10: “ $\mathbf{w}_t \cdot v_1 = 6$ ” should be “ $\mathbf{w}_t \cdot v_1 = 11$ ”

Page 444, line 14: “ $v_2 = (0, -, -1)$ ” should be “ $v_3 = (0, 1, -1)$ ”

Page 473, line -3: Replace “ $\langle x_1^{n_1-1} - 1, \dots, x_m^{n_m-1} - 1 \rangle$ ” with “ $\langle x_1^{n_1} - 1, \dots, x_m^{n_m} - 1 \rangle$ ”

Page 474, line 6: Replace “ $\langle x_1^{n_1-1} - 1, \dots, x_m^{n_m-1} - 1 \rangle$ ” with “ $\langle x_1^{n_1} - 1, \dots, x_m^{n_m} - 1 \rangle$ ”

Page 496, line 7: Replace “of elements” with “of nonzero elements”

Page 496, line -3: Replace “are verified” with “are satisfied”

Page 502, line 10: Replace “ $x_1 x_2^4$ ” with “ $x_1 x_2^3$ ”

Page 553, first column, line -14: Replace “Faugère, C.” with “Faugère, J.-C.”