Errata in the Third Edition of Primes of the Form $x^2 + ny^2$ April 21, 2025

Page 24: In display (2.12), the right-hand side of the inequality should be

$$\sqrt{\frac{-D}{3}}.$$

(This error also appears on page 26 of the second edition.)

Page 50: On line -11, "one more *n* (see Weinberger [**137**])" should be "two more *n* (this follows from Weinberger [**137**])". (This error also appears on page 56 of the second edition.)

Page 98: On the first line of (6.9), " $\sqrt{p_i^*}$ " should be " $\sqrt{p_1^*}$ ". (This error also appears on page 112 of the second edition.)

Page 137: On the final line of part (a) of Exercise 8.1, add the following sentence to the hint: "For the surjectivity of ϕ , use the ideas of part (a) of Exercise 7.29 to show that when \mathfrak{b} is relatively prime to \mathfrak{m} , there is $\gamma \in \mathcal{O}_K$ with $\gamma \equiv 0 \mod \mathfrak{b}$ and $\gamma \equiv 1 \mod \mathfrak{m}$."

Pages 397–398: Replace everything from line -6 on page 397 to line 2 on page 398 with the following:

is exact, where $\iota(\alpha) = [\alpha]$. To prove that ϕ is surjective, choose an element $\alpha \mathcal{O}_K \in I_K(\mathfrak{m}) \cap P_K$; then $\alpha \mathcal{O}_K = \mathfrak{a}\mathfrak{b}^{-1}$, where $\alpha \in K^*$ and $\mathfrak{a}, \mathfrak{b}$ are \mathcal{O}_K -ideals prime to \mathfrak{m} . Since \mathfrak{b} and \mathfrak{m} are relatively prime, the solution to part (a) of Exercise 7.29 gives an isomorphism

$$\mathcal{O}_K/\mathfrak{bm}\simeq \mathcal{O}_K/\mathfrak{b}\times \mathcal{O}_K/\mathfrak{m}.$$

It follows that there is $\gamma \in \mathcal{O}_K$ such that $\gamma \equiv 0 \mod \mathfrak{b}$ and $\gamma \equiv 1 \mod \mathfrak{m}$. Thus $\gamma \in \mathfrak{b}$, so that

$$\gamma \alpha \mathcal{O}_K = \gamma \mathfrak{a} \mathfrak{b}^{-1} = \mathfrak{a}(\gamma \mathfrak{b}^{-1}) \subset \mathfrak{a}(\mathfrak{b} \mathfrak{b}^{-1}) = \mathfrak{a} \subset \mathcal{O}_K,$$

which shows that $\gamma \alpha \in \mathcal{O}_K$. By construction $\gamma \mathcal{O}_K \in P_{K,1}(\mathfrak{m})$, so we get finally $[\alpha \mathcal{O}_K] = [\gamma \alpha \mathcal{O}_K] = \phi([\gamma \alpha]).$