Using Algebraic Geometry, *second edition*

May 8, 2017

Errata:

Page 9, line 4: Replace "$\alpha \cdot \mathbf{w}_1$ and $\alpha \cdot \mathbf{w}_1$" with "$\alpha \cdot \mathbf{w}_1$ and $\beta \cdot \mathbf{w}_1$"

Page 18, line 6: "second polynomial" should be "first polynomial"

Page 28, line 4: Replace "last" with "first"

Page 39, line −7: Replace "$\mathbf{V}(I) \subset \mathbb{C}^n$" with "$\mathbf{V}(I) = \{a \in \mathbb{C}^n : f(a) = 0 \text{ for all } f \in I\}$"

Page 45, part c of Exercise 9: Delete and replace with "Show that $1 = \sum_j (1/p_j(a_j))p_j$."

Page 46, Exercise 11: Add a new part c of the exercise as follows:

c. For readers familiar with the Hermitian inner product $\langle z, w\rangle = \sum_{i=1}^n z_i\overline{w}_i$ for $z, w \in \mathbb{C}^n$, let
$$h_i(x) = \prod_{j\neq i} \langle x - p_i, p_i - p_j\rangle.$$
Show that $g_i(x) = h_i(x)/h_i(p_i)$ satisfies part b.

Page 54, line 5: Replace "$L(x - \sum_j c_j x^{\alpha(j)}) = 0$" with "$L(x^\alpha - \sum_j c_j x^{\alpha(j)}) = 0$"

Page 55, line 2 of Exercise 2: Replace "$x^\alpha > x_1^a$" with "$x^\alpha \geq x_1^a$"

Page 55, line 2 of Exercise 3: Replace "let x^α be" with "let $x^\alpha = x_1^{a_1} \cdots x_n^{a_n}$ be"

Page 65, line 7 after the second display: Replace "$i_1 > \cdots > i_l$" with "$i_1 < \cdots < i_l$"

Page 71, lines 4–8 of the proof of Theorem (5.2): Replace "Hence we will only ... invertible matrix" with "Hence we will only discuss the broad outline of the proof. In the case when I is radical, it is possible to turn the sketch that follows into a rigorous proof."

Page 75, line −3: Replace "$-\text{rem}(p_{i-1}(t), p_{i-2}(t), t)$" with "$-\text{rem}(p_{i-2}(t), p_{i-1}(t), t)$"

Page 75, line −2: Replace "division of p_{i-1} by p_{i-2}" with "division of p_{i-2} by p_{i-1}"

Page 92, line 8: Replace "degrees $d_1, \ldots, d_n$" with "$d_0, \ldots, d_n$"

Page 97, lines 21 and line 24: Replace "Theorem (2.6)" with "Proposition (4.7)"

Page 100, line −2: Replace"Theorem (2.6)" with "Proposition (4.7)"

Page 101, part b of Exercise 10, line 2: Replace "Theorem (2.6)" with "Proposition (4.7)"

Page 102, part c of Exercise 11, line 2: Replace "multiplication by $(-1)^n$" with "multiplication by $(-1)^{n-1}$"

Page 102, part d of Exercise 11: Replace "Theorem (3.5)" with "Theorem (3.4)"

Page 106, line 2 of Exercise 8: Replace "total degree 420" with "total degree 210"

Page 107, line 2 of the proof of Proposition (4.7): Replace "$(n-1)!$ ways" with "$n!$ ways"

Page 108, line 2 of Exercise 11: Replace "D_3' " with "D_2' "

Page 109, line 3: Replace "Exercise 10" with "Exercise 11"

Page 113, part d of Exercise 22: Replace part d with "Use part c to show that the determinant in (2.8) vanishes whenever $F_0 = F_1 = F_2 = 0$ has a nontrivial solution."

Page 120, line 6 after display (5.12): Replace "$u_1 = \cdots = u_n = 0$" with "$u_1 = \cdots = u_{n-1} = 0$"

Page 123, line 11: Replace "$A = \mathbb{C}(u)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_n\rangle$" with "$A = \mathbb{C}(u_0)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_{n-1}\rangle$"

Page 123, line 19: Replace "$\widehat{A} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_n\rangle$" with "$\widehat{A} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_{n-1}\rangle$"

Page 125, last display: Replace "$F_0 = \cdots = F_n = 0$" with "$F_1 = \cdots = F_n = 0$"

Page 126, line 8: Replace "$f_0 = \cdots = f_n = 0$" with "$f_1 = \cdots = f_n = 0$"

Page 129, line −2: Replace "$x^\alpha/x_i^{d_i}$ has degree $\leq d - d_i$" with "$x^\beta/x_i^{d_i}$ has degree $\leq d - d_i$"

Page 131, lines 1 and 2 following second-to-last display: Replace "Exercise 12 of Chapter 2, §4" with "Exercise 12 of Chapter 2, §2"

Page 134, line 2 of Exercise 3: Replace "$(u_0, u_1, u_2, u_3) = (0, 1, 0, 0)$" with "$(u_0, u_1, u_2) = (0, 1, 0)$"

Page 137, line 14: Replace "both these types" with "both types"

Page 138, line −12: Replace "if $N \neq M$ is an ideal in R with $M \subset N \subset R$," with "if $N \not\subset M$ is an ideal in R,"

Page 138, lines −10 to −8: Replace "Therefore M ... contained in M." with "Therefore, every proper ideal of R is contained in M. Hence M is maximal and is the only maximal ideal of R."

Page 142, line 3: Replace "$\sum_{n\geq 0} f_n(x)$" with "$\sum_{m\geq 0} f_m(x)$"

Page 143, line 4: Replace "$f_n(x) = \sum_{\substack{\alpha\in\mathbb{Z}^n_{\geq 0}\\ |\alpha|=n}} c_\alpha x^\alpha$" with "$f_m(x) = \sum_{\substack{\alpha\in\mathbb{Z}^n_{\geq 0}\\ |\alpha|=m}} c_\alpha x^\alpha$"

Page 143, line 5: Replace the display with $h_m = f_m g_0 + f_{m-1} g_1 + \cdots + f_0 g_m$."

Page 143, line 7: Replace "n" with "m"

Page 147, part a of Exercise 2: Replace "$\mathbf{V}(x^2 - 2x + y^2, x^2 - 4x + 4y^4)$" with "$\mathbf{V}(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$"

Page 147, part b of Exercise 2: Replace "$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^4\rangle$" with "$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^2\rangle$"

Page 149, line −18: Replace "note that that" with "note that the"

Page 152, line 6: Replace "Proposition (5.9)" with "Proposition (5.15)"

Page 152, line 3 of Exercise 3: Replace "$f_2 = 6y - x^3 + 9x$," with "$f_2 = 6y - x^3 + 9x = 0$,"

Page 152, line 1 of part e of Exercise 3: Replace "$\mathrm{Res}(f_1, f_2, y)$" with "$\mathrm{Res}^y(f_1, f_2)$ as defined in (5.14) of Chapter 3"

Page 152, line 4 of part e of Exercise 3: Replace "$\mathrm{Res}(f_1, f_2, x)$" with "$\mathrm{Res}^x(f_1, f_2)$"

Page 156, part c of Exercise 10, line 4: Replace "$A_i \Leftrightarrow f(p) = \lambda$" with "$A_i \Leftrightarrow f(p_i) = \lambda$"

Page 160, line -2: Replace "$\ker(M) \cap \mathbb{Z}^n_{\geq 0}$" with "$\ker(M) \cap \mathbb{Z}^n$"

Page 161, line -3: Replace "$S = \{1 + g \ : \ \mathrm{LT}(g) < 1\}$" with "$S = \{1 + g \ : \ g = 0 \text{ or } \mathrm{LT}(g) < 1\}$"

Page 163, line 1 of part a of Exercise 5: Replace "let $h \in A$" with "let $h \in \mathrm{Loc}_>(A)$"

Page 163, line 1 of part b of Exercise 5: Replace "Let $r \in R$" with "Let $r \in \mathrm{Loc}_>(A)$"

Page 166, lines -20 and -18: Replace "$t^a > t^{a'} x^\beta$" with "$t^a >' t^{a'} x^\beta$" (twice)

Page 171, line 3 of part c of Exercise 8: Replace "$1/(1 + h)$" with "$1/(1 + g)$"

Page 172, line 2 of Exercise 11: Replace "(for local orders)" with "(for degree-anticompatible orders)"

Page 172, line 1 of part a of Exercise 11: Replace "Let $>$ be a local order" with "Let $>$ be a degree-anticompatible order"

Page 193, line -2: Replace "When M and N are free modules," with "When $M = R^l$ and $N = R^m$,"

Page 197, line -10: Besides the 1994 paper [PW] by Park and Woodburn, we should also mention two other papers that deal with algorithmic aspects of the Quillen-Suslin result:

L. Caniglia, G. Cortiñas, S. Danón, J. Heintz, T. Krick and P. Solernó (working group N. Fitchas), *Algorithmic aspects of Suslin's proof of Serre's conjecture*, Comput. Complexity **3** (1993), 31–55

N. Fitchas and A. Galligo, *Nullstellensatz effectif et conjecture de Serre (théorème de Quillen-Suslin) pour le calcul formel*, Math. Nachr. **149** (1990), 231–253.

Page 203, line 1: Replace "Let $\varphi : M \to N$." with "Let $\varphi : M \to N$ be an R-module homomorphism."

Page 199, line 4: "Equivalently. we think" should be "Equivalently, we think"

Page 203, line 2 of part a of Exercise 23: Replace "$\{af : a \in I, f \in M\}$" with "$\{\sum_{i=i}^{\ell} a_i f_i : a_i \in I, f_i \in M \text{ for } i = 1, \ldots, \ell\}$".

Page 203, line -16: Replace "We let $R = k[x, y]$" with "Let $R = k[x, y]$, where k is a field of characteristic different from 2,"

Page 203, lines -9 to -1: Delete and replace with the following:

a. Verify that $\mathbf{f} = (f_1, f_2, f_3)^T = (1, -x/2, -1/2)^T \in R^3$ satisfies $(1 + x)f_1 + (1 - y)f_2 + (x + xy)f_3 = 1$.

b. Let I be the 3×3 identity matrix. Verify that the columns $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$ of the matrix $I - \mathbf{f} \cdot A$ span $\ker A$. Hint: If $A\tilde{\mathbf{f}} = 0$, then $\tilde{\mathbf{f}} = (I - \mathbf{f} \cdot A)\tilde{\mathbf{f}}$ is a linear combination of the columns of $I - \mathbf{f} \cdot A$.

c. Show that $\{\mathbf{g}_1, \mathbf{g}_2\}$ is a basis of $\ker A$. (Unfortunately, the result of part c is special to the choice of $\mathbf{f}$ made in part a. If $\mathbf{f}$ is an arbitrary solution of $A\mathbf{f} = 1$, then the first two columns of $I - \mathbf{f} \cdot A$ need not give a basis of the kernel.)

Page 205, line 5: Replace "show that M" with "then M"

Page 205, line 6: Replace "R^l to R^m" with "R^m to R^l"

Page 210, lines −9 and −8: Replace "(see Exercise 5 below)" with "(see Exercise 11 of §3)"

Page 215, line 10: Replace "$M_{M+\ell}$" with "$M_{N+\ell}$"

Page 224, line 12: Replace "Exercise 1" with "Exercise 2"

Page 227, line −2: Replace "$(AG \;\; I_t - AB)$" with "$(AD \;\; I_t - AB)$".

Page 231, line 2: Replace "R^{n+t+s}" with "R^{m+t+s}"

Page 237, line −6: Replace "Hence" with "If $s > 1$, then"

Page 237, line −2: Add the sentence "If $s = 1$, then $(1 - a_1)f_1 = 0$. This implies $f_1 = 0$, which contradicts $M \neq 0$.

Page 239, line 1: Replace "matrix of $M/\mathfrak{m}M$." with "matrix of $M/\mathfrak{m}M$?"

Page 240, line 4: Replace "have have" with "have"

Page 248, line −9: Replace "*Exercise 12*" with "*Exercise 28*"

Page 253, line −6: Replace with "$M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle$"

Page 254, line 1: Replace with "`M = ideal(y*z-x*w,y^3-x^2*z,x*z^2-y^2*w,z^3-y*w^2)`"

Page 263, line 1: Replace "from (1.8)" with "from (1.7)"

Page 263, line 6: Replace "see (1.16)" with "(see (1.14))"

Page 267, line 1 of **(3.3) Proposition**: Replace "be submodule" with "be a submodule"

Page 269, line 1 of Exercise 3: Replace "finitely generated" with "finitely generated graded"

Page 270, line 6: Replace with "$M = \langle yz - xw, y^3 - x^2z, xz^2 - y^2w, z^3 - yw^2 \rangle$"

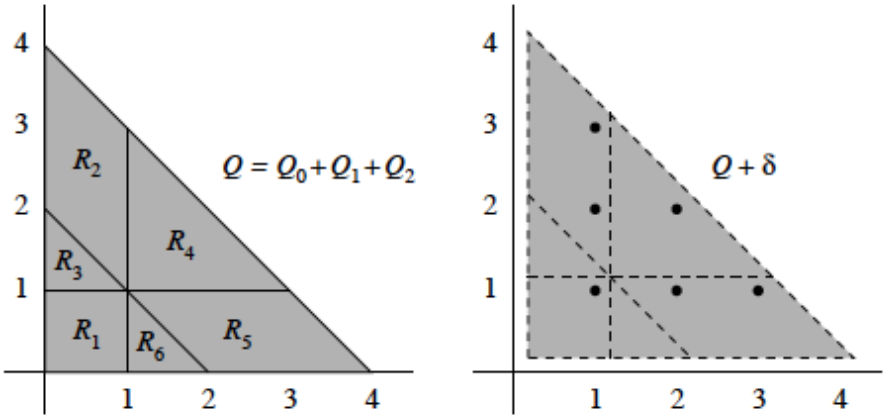
Page 270, line 9: Replace with "$R(-2) \oplus R(-3)^3 \to R$"

Page 308, line before Exercise 4: Add a new sentence "We also regard Q as a face of itself."

Page 308, line following Exercise 4: Replace "Every face" with "Every proper face"

Page 327, line 9: In the statement of Theorem (3.13), replace "$\mathcal{A} = \{m_1, \ldots, m_l\} \subset \mathbb{Z}^n_{\geq 0}$" with "$\mathcal{A} = \{m_1, \ldots, m_l\} \subset \mathbb{Z}^n$"

Page 365, Figure 7.9: The figure is wrong. Here is the correct figure.



Page 431, line −6: The left-hand side of the equation should be "$\{\underline{x^2} - y, \underline{yz} + xz - y^2\}$"

Page 433, line 1: Replace "that w" with "that $\mathbf{w}$"

Page 444, line 10: "$\mathbf{w}_t \cdot v_1 = 6$" should be "$\mathbf{w}_t \cdot v_1 = 11$"

Page 444, line 12: "$v_2 = (0, -, -1)$" should be "$v_3 = (0, 1, -1)$"

Page 553, first column, line −14: Replace "Faugère, C." with "Faugère, J."