Using Algebraic Geometry, second edition

May 8, 2017

Errata:

Page 9, line 4: Replace “α · w_1 and α · w_1” with “α · w_1 and β · w_1”

Page 18, line 6: “second polynomial” should be “first polynomial”

Page 28, line 4: Replace “last” with “first”

Page 39, line 7: Replace “V(I) ⊂ C^n” with “V(I) = \{a ∈ C^n : f(a) = 0 for all f ∈ I\}”

Page 45, part c of Exercise 9: Delete and replace with “Show that 1 = \sum_j (1/p_j(a_j))p_j.”

Page 46, Exercise 11: Add a new part c of the exercise as follows:

c. For readers familiar with the Hermitian inner product \langle z, w \rangle = \sum_{i=1}^{n} z_i \overline{w_i} for z, w ∈ C^n, let

\[ h_i(x) = \prod_{j \neq i} (x - p_i, p_i - p_j). \]

Show that \[ g_i(x) = h_i(x)/h_i(p_i) \] satisfies part b.

Page 54, line 5: Replace “L(x - \sum_j c_j x^{α(j)}) = 0” with “L(x^{α} - \sum_j c_j x^{α(j)}) = 0”

Page 55, line 2 of Exercise 2: Replace “x^{α} > x_1^{α}” with “x^{α} ≥ x_1^{α}”

Page 65, line 7 after the second display: Replace “i_1 > ... > i_l” with “i_1 < ... < i_l”

Page 71, lines 4–8 of the proof of Theorem (5.2): Replace “Hence we will only ... invertible matrix” with “Hence we will only discuss the broad outline of the proof. In the case when I is radical, it is possible to turn the sketch that follows into a rigorous proof.”

Page 75, line 3: Replace “− rem(p_{i-1}(t), p_{i-2}(t), t)” with “− rem(p_{i-2}(t), p_{i-1}(t), t)”

Page 75, line 2: Replace “division of p_{i-1} by p_{i-2}” with “division of p_{i-2} by p_{i-1}”

Page 92, line 8: Replace “degrees d_1, ..., d_n” with “d_0, ..., d_n”

Page 97, lines 21 and line 24: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 100, line 2: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 101, part b of Exercise 10, line 2: Replace “Theorem (2.6)” with “Proposition (4.7)”

Page 102, part c of Exercise 11, line 2: Replace “multiplication by (−1)^n” with “multiplication by (−1)^{n−1}”

Page 102, part d of Exercise 11: Replace “Theorem (3.5)” with “Theorem (3.4)”

Page 106, line 2 of Exercise 8: Replace “total degree 420” with “total degree 210”

Page 107, line 2 of the proof of Proposition (4.7): Replace “(n − 1)! ways” with “n! ways”

Page 108, line 2 of Exercise 11: Replace “D_3’” with “D_2’”
Page 109, line 3: Replace “Exercise 10” with “Exercise 11”

Page 113, part d of Exercise 22: Replace part d with “Use part c to show that the determinant in (2.8) vanishes whenever $F_0 = F_1 = F_2 = 0$ has a nontrivial solution.”

Page 120, line 6 after display (5.12): Replace “$u_1 = \cdots = u_n = 0$” with “$u_1 = \cdots = u_{n-1} = 0$”

Page 123, line 11: Replace “$A = \mathbb{C}(u)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_n \rangle$” with “$\hat{A} = \mathbb{C}(u_0)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_{n-1} \rangle$”

Page 123, line 19: Replace “$\hat{A} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_n \rangle$” with “$\hat{A} = \mathbb{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_{n-1} \rangle$”

Page 125, last display: Replace “$F_0 = \cdots = F_n = 0$” with “$F_1 = \cdots = F_n = 0$”

Page 126, line 8: Replace “$f_0 = \cdots = f_n = 0$” with “$f_1 = \cdots = f_n = 0$”

Page 129, line –2: Replace “$x^\alpha/x_i^d$ has degree $\leq d - d_i$” with “$x^\beta/x_i^d$ has degree $\leq d - d_i$”

Page 131, lines 1 and 2 following second-to-last display: Replace “Exercise 12 of Chapter 2, §4” with “Exercise 12 of Chapter 2, §2”

Page 134, line 2 of Exercise 3: Replace “$(u_0, u_1, u_2, u_3) = (0, 1, 0, 0)$” with “$(u_0, u_1, u_2) = (0, 1, 0)$”

Page 137, line 14: Replace “both these types” with “both types”

Page 138, line –12: Replace “if $N \neq M$ is an ideal in $R$ with $M \subset N \subset R$,” with “if $N \not\subset M$ is an ideal in $R$,”

Page 138, lines –10 to –8: Replace “Therefore $M \ldots$ contained in $M$.” with “Therefore, every proper ideal of $R$ is contained in $M$. Hence $M$ is maximal and is the only maximal ideal of $R$.”

Page 142, line 3: Replace “$\sum_{n \geq 0} f_n(x)$” with “$\sum_{m \geq 0} f_m(x)$”

Page 143, line 4: Replace “$f_n(x) = \sum_{\alpha \in \mathbb{Z}_{\geq 0}^n, |\alpha| = n} c_\alpha x^\alpha$” with “$f_m(x) = \sum_{\alpha \in \mathbb{Z}_{\geq 0}^n, |\alpha| = m} c_\alpha x^\alpha$”

Page 143, line 5: Replace the display with $h_m = f_m g_0 + f_{m-1} g_1 + \cdots + f_0 g_m$.

Page 143, line 7: Replace “$n$” with “$m$”

Page 147, part a of Exercise 2: Replace “$\nabla(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$” with “$\nabla(x^2 - 2x + y^2, x^2 - 4x + 4y^2)$”

Page 147, part b of Exercise 2: Replace “$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^4 \rangle$” with “$\langle x^2 - 2x + y^2, x^2 - 4x + 4y^4 \rangle$”

Page 149, line –18: Replace “note that that” with “note that the”

Page 152, line 6: Replace “Proposition (5.9)” with “Proposition (5.15)”

Page 152, line 3 of Exercise 3: Replace “$f_2 = 6y - x^3 + 9x$, with “$f_2 = 6y - x^3 + 9x = 0$,”

Page 152, line 1 of part e of Exercise 3: Replace “$\text{Res}(f_1, f_2, y)$” with “$\text{Res}^y(f_1, f_2)$ as defined in (5.14) of Chapter 3’”
Page 152, line 4 of part e of Exercise 3: Replace “Res\((f_1, f_2, x)\)” with “\(\text{Res}^x(f_1, f_2)\)”

Page 156, part c of Exercise 10, line 4: Replace “\(A_i \iff f(p) = \lambda\)” with “\(A_i \iff f(p_i) = \lambda\)”

Page 160, line –2: Replace “\(\ker(M) \cap \mathbb{Z}^n_\geq\)” with “\(\ker(M) \cap \mathbb{Z}^n\)”

Page 161, line –3: Replace “\(S = \{1 + g : \text{LT}(g) < 1\}\)” with “\(S = \{1 + g : g = 0 \text{ or } \text{LT}(g) < 1\}\)”

Page 163, line 1 of part a of Exercise 5: Replace “let \(h \in A\)” with “let \(h \in \text{Loc}_>(A)\)”

Page 163, line 1 of part b of Exercise 5: Replace “Let \(r \in \mathbb{R}\)” with “Let \(r \in \text{Loc}_>(\mathbb{R})\)”

Page 166, lines –20 and –18: Replace “\(t^a > t^a' \cdot x^\beta\)” with “\(t^a > t^a' \cdot x^\beta\)” (twice)

Page 171, line 3 of part c of Exercise 8: Replace “\(1/(1 + h)\)” with “\(1/(1 + g)\)”

Page 172, line 2 of Exercise 11: Replace “(for local orders)” with “(for degree-anticompatible orders)”

Page 172, line 1 of part a of Exercise 11: Replace “Let > be a local order” with “Let > be a degree-anticompatible order”

Page 193, line –2: Replace “When \(M\) and \(N\) are free modules,” with “When \(M = R^l\) and \(N = R^m\),”

Page 197, line –10: Besides the 1994 paper [PW] by Park and Woodburn, we should also mention two other papers that deal with algorithmic aspects of the Quillen-Suslin result:


Page 203, line 1: Replace “Let \(\varphi : M \to N\)” with “Let \(\varphi : M \to N\) be an \(R\)-module homomorphism.”

Page 199, line 4: “Equivalently, we think” should be “Equivalently, we think”

Page 203, line 2 of part a of Exercise 23: Replace “\(\{af : a \in I, f \in M\}\)” with “\(\{\sum_{i=1}^{\ell} a_i f_i : a_i \in I, f_i \in M \text{ for } i = 1, \ldots, \ell\}\)”.

Page 203, line –16: Replace “We let \(R = k[x, y]\)” with “Let \(R = k[x, y]\), where \(k\) is a field of characteristic different from 2,”

Page 203, lines –9 to –1: Delete and replace with the following:

a. Verify that \(f = (f_1, f_2, f_3)^T = (1, -x/2, -1/2)^T \in R^3\) satisfies \((1 + x)f_1 + (1 - y)f_2 + (x + xy)f_3 = 1\).

b. Let \(I\) be the \(3 \times 3\) identity matrix. Verify that the columns \(g_1, g_2, g_3\) of the matrix \(I - f \cdot A\) span \(\ker A\). Hint: If \(Af = 0\), then \(\tilde{f} = (I - f \cdot A)f\) is a linear combination of the columns of \(I - f \cdot A\).
c. Show that $\{g_1, g_2\}$ is a basis of $\ker A$. (Unfortunately, the result of part c is special to the choice of $f$ made in part a. If $f$ is an arbitrary solution of $A f = 1$, then the first two columns of $I - f \cdot A$ need not give a basis of the kernel.)

Page 205, line 5: Replace “show that $M$” with “then $M$”

Page 205, line 6: Replace “$R^t$ to $R^m$” with “$R^m$ to $R^t$”

Page 210, lines −9 and −8: Replace “(see Exercise 5 below)” with “(see Exercise 11 of §3)”

Page 227, line −2: Replace “$(AG I_t - AB)$” with “$(AD I_t - AB)$”.

Page 231, line 2: Replace “$R^{n+t+s}$” with “$R^{m+t+s}$”

Page 237, line −6: Replace “Hence” with “If $s > 1$, then”

Page 237, line −2: Add the sentence “If $s = 1$, then $(1 - a_1)f_1 = 0$. This implies $f_1 = 0$, which contradicts $M \neq 0$.

Page 239, line 1: Replace “matrix of $M/\mathfrak{m}M.$” with “matrix of $M/\mathfrak{m}M$?”

Page 240, line 4: Replace “have have” with “have”

Page 248, line −9: Replace “Exercise 12” with “Exercise 28”

Page 253, line −6: Replace with “$M = \langle y z - x w, y^3 - x^2 z, x z^2 - y^2 w, z^3 - y w^2 \rangle$”

Page 254, line 1: Replace with “$M = \text{ideal}(y z - x w, y^3 - x^2 z, x z^2 - y^2 w, z^3 - y w^2)$”

Page 263, line 1: Replace “from (1.8)” with “from (1.7)”

Page 263, line 6: Replace “see (1.16)” with “(see (1.14))”

Page 267, line 1 of (3.3) Proposition: Replace “be submodule” with “be a submodule”

Page 269, line 1 of Exercise 3: Replace “finitely generated” with “finitely generated graded”

Page 270, line 6: Replace with “$M = \langle y z - x w, y^3 - x^2 z, x z^2 - y^2 w, z^3 - y w^2 \rangle$”

Page 270, line 9: Replace with “$R(-2) \oplus R(-3)^3 \to R$”

Page 308, line before Exercise 4: Add a new sentence “We also regard $Q$ as a face of itself.”

Page 308, line following Exercise 4: Replace “Every face” with “Every proper face”

Page 327, line 9: In the statement of Theorem (3.13), replace “$A = \{m_1, \ldots, m_l\} \subset \mathbb{Z}_{\geq 0}^n$” with “$A = \{m_1, \ldots, m_l\} \subset \mathbb{Z}^n$”

Page 365, Figure 7.9: The figure is wrong. Here is the correct figure.
Page 431, line −6: The left-hand side of the equation should be “\(\{x^2 - y, yz + xz - y^2\}\)”

Page 433, line 1: Replace “that \(w\)” with “that \(\mathbf{w}\)”

Page 444, line 10: “\(\mathbf{w}_t \cdot \mathbf{v}_1 = 6\)” should be “\(\mathbf{w}_t \cdot \mathbf{v}_1 = 11\)”

Page 444, line 12: “\(\mathbf{v}_2 = (0, -, -1)\)” should be “\(\mathbf{v}_3 = (0, 1, -1)\)”

Page 553, first column, line −14: Replace “Faugère, C.” with “Faugère, J.”