Ideals, Varieties and Algorithms, *fourth edition*

Errata for the fourth edition as of May 24, 2017

Page 23, part (c) of Exercise 5:"Adapt the argument given at the end of the section" should be "Adapt the argument used for the circle $x^2 + y^2 = 1$"

Page 24, line 1: "adapt the argument given at the end of the section" should be "adapt the argument used for the circle $x^2 + y^2 = 1$"

Page 37, line 2 of part (c) of Exercise 16: "Binomial Theorem" should be "binomial theorem"

Page 39, line 6 of the paragraph beginning "To see why this algorithm works": "By (5)," should be "By (1),"

Page 53, line 2 of Exercise 5: "affine algebraic varieties" should be "affine varieties"

Page 63, bottom display, second line underneath $\overline{}$: "$xy^2 - x$" should be "$x^2y - x$"

Page 63, bottom display, fourth line underneath $\overline{}$: "$x^2y - y$" should be "$xy^2 - y$"

Page 66, lines -5 and -6: "[using condition (ii) of the definition of a monomial order]" should be "[using Lemma 8 of §2]"

Page 81, line 1 of Exercise 2: "$\text{LT}(I)$" should be "$\langle \text{LT}(I) \rangle$"

Page 82, last line of Exercise 13: "Exercise 14 of Chapter 1, §4" should be "Proposition 8 of Chapter 1, §4"

Page 89, part (d) of Exercise 5: "$z^2 - 3^z$" should be "$z^2 - 3z$"

Page 93, line -3: "$\langle \text{LT}(G \setminus \{p\}) \rangle$" should be "$\langle \text{LT}(G \setminus \{g\}) \rangle$"

Page 96, line 4 of part (b) of Exercise 10: "$g_j = x_t + D$" should be "$g_j = x_\ell + D$"

Page 96, line 4 of part (b) of Exercise 10: "$S(g_i, g_j) = x_tC - x_sD$" should be "$S(g_i, g_j) = x_\ell C - x_sD$"

Page 96, line 2 of part (a) of Exercise 12: "is not divisible by" should be "has leading term not divisible by"

Page 97, line 3 of Exercise 14: "$\dfrac{x_j - a_j}{a_i - a_j}$" should be "$\dfrac{x - a_j}{a_i - a_j}$"

Page 105, lines 13–16: Replace $xy + 1$ with $xy - 1$ in three places and $-x - y$ with $-x + y$ in two places. Thus the lines should be as follows:

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§3. If we divide $f = xy^2 - x$ by $G = (xy - 1, y^2 - 1)$, the division algortihm gives

$$xy^2 - x = y \cdot (xy - 1) + 0 \cdot (y^2 - 1) + (-x + y)$$

so that $\overline{f}^G = -x + y \neq 0$. Yet we can also write

$$xy^2 - x = 0 \cdot (xy - 1) + x \cdot (y^2 - 1),$$

Page 107, line −4: "$f_3 = xz + y - z + 1$" should be "$f_3 = xz - x + y + 1$"

Page 108, Exercise 1: "$f_3 = xz + y - z + 1$" should be "$f_3 = xz - x + y + 1$"

Page 110, Lemma 4: The statement of the lemma should be changed to the following:

Lemma 4. *Every element of $S(F)$ can be written as a sum of homogeneous elements of $S(F)$. Furthermore, this decomposition is unique.*

Page 115, line 12 "by Lemma 2" should be "by Lemma 2 of §9"

Page 136, line 20: "$(t, u, x, y, z) \in \mathbf{V}(I) \subseteq \mathbb{R}^5$" should be "$(t, u, x, y, z) \in \mathbf{V}(I) \subseteq \mathbb{C}^5$"

Page 140, Exercise 3: "t^2 is always positive" should be "t^2 is always ≥ 0"

Page 142, part (a) of Exercise 11: In two places, replace F by G

Page 142, part (b) of Exercise 11: Replace F by G

Page 142, line 3 of Exercise 13: "$x_i = f_i(t)/g_i(t)$" should be "$x_\ell = f_\ell(t)/g_\ell(t)$"

Page 142, line 4 of Exercise 13: "each i, $f_i(t)$ and $g_i(t)$" should be "each ℓ, $f_\ell(t)$ and $g_\ell(t)$"

Page 153, line 1: "straight lines" should be "straight line"

Page 153, line 4: "These lines have" should be "This line has"

Page 154, part (c) of Exercise 15: The x-coordinate of the second displayed point should be

$$\pm\tfrac{1}{2}\sqrt{15 + 6\sqrt[3]{2} - 12\sqrt[3]{4}}$$

Page 160, part (b) of Exercise 4: "$g_{\text{o}} = g_3$" should be "$g_{\text{o}} = g_2$"

Page 167, line −16: Replace "It follows that" with "(Lemma 3 and Proposition 5 apply to f, g since their coefficients lie in the field $k(x_2, \ldots, x_n)$.) It follows that"

Page 169, line −7: "$u_1(x_1)$" should be "$u(x_1)$"

Page 170, line 1: "$\langle f_i, f_* \rangle$" should be "$\langle f_i, f^* \rangle$"

Page 170, part (c) of Exercise 3: "part (a) is still true but part (b) can fail" should be "parts (a) and (b) are still true"

Page 180, line 13: "must have $f_i(a_1, \ldots, a_n) = 0$" should be "must have $f_i(a_1, \ldots, a_n) \neq 0$"

Page 181, lines 1 and 2 of Exercise 10: "$\mathbb{R}[x, y]$" should be "$\mathbb{R}[x, y, z]$" in two places.

Page 181, line 4 of Exercise 10: "same for $\mathbb{R}[x]$" should be "same for $\mathbb{R}[x]$ and $\mathbb{R}[x, y]$"

Page 187, line -3: "$a_1 \dfrac{\partial f_i}{\partial x_j} h_i$" should be "$a_i \dfrac{\partial f_i}{\partial x_j} h_i$"

Page 190, line 15: "We have sketched $\mathbf{V}(I)$ and $\mathbf{V}(J)$ on the next page" should be "We have sketched $\mathbf{V}(I)$ and $\mathbf{V}(J)$ below"

Page 196, line 4: "principal ideals is principal)" should be "two principal ideals is principal)"

Page 199, lines 1–2 of **Definition 2**: "smallest affine algebraic variety" should be "smallest affine variety"

Page 201, line 12: "*varieties k^n*" should be "*varieties in k^n*"

Page 206, Exercise 7: "and the Exercise 4" should be "and Exercise 4"

Page 219, line -18: "is projection" should be "be the projection"

Page 221, **Corollary 3**: "With the same notation" should be "With k algebraically closed and the same notation"

Page 222, line 4: "for all i" should be "for all such i"

Page 222, line 6: "If follows that" should be "It follows that"

Page 222, line immediately before **Proposition 5**: "what works for V" should be "that works for V"

Page 222, line 4 of **Proposition 5**: "$\pi_l(\mathbf{V}(I^{(i)})$" should be "$\pi_l(\mathbf{V}(I^{(i)}))$"

Page 222, line 5 of **Proposition 5**: "a variety contained in V" should be "a variety contained in $\mathbf{V}(I_l)$"

Page 223, line -17: "$W \subsetneq \mathbf{V}(I)$" should be "$W \subsetneq \mathbf{V}(I_l)$"

Page 223, line -9: "fails for I, $\mathbf{V}(I)\backslash$" should be "fails for I, $\mathbf{V}(I_l)\backslash$"

Page 223, line -8: "by Proposition 4" should be "by Proposition 4 (we can assume G is reduced)"

Page 224, second display: The display should be as follows:

$$\mathbf{V}(I_1) \setminus \mathbf{V}(c_1) = \mathbb{C} \setminus \mathbf{V}(y) = \mathbb{C} \setminus \{0\} \subseteq \pi_1(\mathbf{V}(I)) \subseteq \mathbf{V}(I_1) = \mathbb{C}.$$

Page 226, two lines below display (2): "$W_2 \setminus Z_2 \subset \pi_l(V_1)$" should be "$W_2 \setminus Z_2 \subseteq \pi_l(V_1)$"

Page 231, line 9: "**EXERCISES FOR §9**" should be "**EXERCISES FOR §8**"

Page 231, part (b) of Exercise 11: "Exercise 4" should be "Exercise 6"

Page 232, line 2: "The table on the next page" should be "The table below"

Page 255, line 12: "polynomials G_{i-1}" should be "polynomials in G_{i-1}"

Page 257, part (e) of Exercise 11: "we developed in Chapter 1" should be "we developed in Chapter 1, §2"

Page 260, lines 2–4: These three lines

(iii) is proved in the same way as Theorem 11 of Chapter 4, §5. □

When k is algebraically closed, the Weak Nullstellensatz also holds in $k[V]$. You will prove this in Exercise 16.

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should be replaced with the following:

(iii) is proved by first showing that the Weak Nullstellensatz also holds in $k[V]$. You will prove this in Exercise 16. From here, one proceeds in the same way as Theorem 11 of Chapter 4, §5. □

Page 271, line following second display: "$\mathbf{V}_W(a^2 - b^2 + 4)$" should be "$\mathbf{V}_W(y^2 - z^2 + 4)$"

Page 272, second paragraph of the proof of **Proposition 6**: In two places, "$\mathbf{V}(f_i g_i' - f_i' g_i)$" should be "$\mathbf{V}_V(f_i g_i' - f_i' g_i)$"

Page 278, line −6: "since we do not the ideal" should be "since we do not want the ideal"

Page 279, display (1): Replace the display with

$$s^\ell + c_1 s^{\ell-1} + \cdots + c_\ell = 0, \quad c_1, \ldots, c_\ell \in R. \tag{1}$$

Page 279, line −10: "$a_{i\ell} s_{i\ell}$" should be "$a_{i\ell} s_\ell$"

Page 279, line −7: "the coefficient of x is" should be "the coefficient of x^ℓ is"

Page 279, the last display should be:

$$\det(A - xI_\ell) = (-1)^\ell (x^\ell + c_1 x^{\ell-1} + \cdots + c_\ell).$$

Page 279, line −4: "$a_i \in R$" should be "$c_i \in R$"

Page 280, line -13: "(ii) $\Rightarrow$ (iii)" should be "(i) $\Rightarrow$ (iii)"

Page 280, line -9: "divide f by G" should be "divide f by a Gröbner basis G"

Page 282, seven lines below the figure: "the Zariski closure" should be "is the Zariski closure"

Page 282, line -6: "finite over $k[y]$" should be "finite over $k[y_1, \ldots, y_m]$"

Page 284, line 1: "The surprise that" should be "The surprise is that"

Page 284, line 10: "algebraically independently" should be "algebraically independent"

Page 288, line 2 of Exercise 6: "means geometrically" should be "means geometrically when k is algebraically closed"

Page 289, last line of part (a) of Exercise 17: "$\phi(\mathbf{V}(J))$" should be "$\pi(\mathbf{V}(J))$"

Page 291, line 8: The correct hyphenation is "re-searchers"

Page 294, line below second display: "$C = U \times V$" should be "$\mathcal{C} = U \times V$"

Page 298, two lines below third display: The correct hyphenation is "ex-ercises"

Page 300, first display: "$f(\theta_1 + \theta_2 + \theta_3)$" should be "$f(\theta_1, \theta_2, \theta_3)$"

Page 300, paragraph beginning "We will next discuss": Insert a blank line between the beginning of this paragraph and the end of the previous paragraph.

Page 302, line 2: "$\mathcal{J} = \mathbf{V}(x_1^2 + y_1^2 - 1, x_2^2 + y_2^2 - 1, x_3^2 + y_3^2 - 1)$" should be "$\mathcal{J} = \mathbf{V}(c_1^2 + s_1^2 - 1, c_2^2 + s_2^2 - 1, c_3^2 + s_3^2 - 1)$"

Page 302, two lines below (7): "$V = \mathbf{V}(x_1^2 + y_1^2 - 1, x_2^2 + y_2^2 - 1, x_3^2 + y_3^2 - 1)$" should be "$V = \mathbf{V}(c_1^2 + s_1^2 - 1, c_2^2 + s_2^2 - 1, c_3^2 + s_3^2 - 1)$"

Page 303, Exercise 2: The correct hyphenation is "co-ordinates"

Page 304, part (a) of Exercise 9: "result of part (c)" should be "result of part (e)"

Page 305, line 12: "in equation (7) of §2" should be "in equation (6) of §2"

Page 305, line 1 of (2): "$\dfrac{2bl_2l_3}{2l_2(a^2+b^2)}s_2$" should be "$\dfrac{bl_2l_3}{l_2(a^2+b^2)}s_2$"

Page 305, line 2 of (2): "$\dfrac{2al_2l_3}{2l_2(a^2+b^2)}s_2 +$ " should be "$\dfrac{al_2l_3}{l_2(a^2+b^2)}s_2 -$ "

Page 306, line 1 of (3): "$\dfrac{2b}{2(a^2+b^2)}s_2$" should be "$\dfrac{b}{a^2+b^2}s_2$"

Page 306, line 2 of (3): "$\dfrac{2a}{2(a^2+b^2)}s_2 +$ " should be "$\dfrac{a}{a^2+b^2}s_2 -$ "

Page 307, line -3: "when $a^2+b^2<4$," should be "when $0<a^2+b^2<4$,"

Page 308, line 8 of the subsection *Specialization of Gröbner Bases*: "$k[x_1,\dots,x_m,t_1,\dots,t_m]$" should be "$k[x_1,\dots,x_n,t_1,\dots,t_m]$"

Page 309, two lines below display: "a^2+b^2 and $a^2+b^2-l_2^2-l_3^2$ are nonzero" should be "a^2+b^2, $a^2+b^2-l_2^2-l_3^2$ and $l_2^2-l_3^2$ are nonzero"

Page 309, nine lines below display: "concept of a" should be "the concept of a"

Page 309, line -4: "to be Gröbner cover" should be "to be a Gröbner cover"

Page 312, line 3: "We have" should be "When $l_2=l_3=1$, we have"

Page 315, part (c) of Exercise 5: "the choice of $t\in\mathbb{R}$." should be "the choice of $t\in\mathbb{R}$?"

Page 316, line 5: "$B_{ij}\in k(\mathbf{t})[\mathbf{x}]$" should be "$B_{ji}\in k(\mathbf{t})[\mathbf{x}]$"

Page 316, part (c) of Exercise 7: Replace the hint with "Hint: The monomial orders for $k(\mathbf{t})[\mathbf{x}]$ and $k[\mathbf{x}]$ are the same—the parameters t_j are "constants" as far as the ordering is concerned. Theorem 6 of Chapter 2, §9 will be useful.

Page 316, lines 7 and 8 of Exercise 8: Delete these lines and replace them with the following:

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nonzero polynomials F_i and G_j in $k[\mathbf{t}]$, we get

$$\tilde{f}_i = F_i f_i,\ \tilde{g}_j = G_j g_j \in k[\mathbf{x},\mathbf{t}].$$

Page 316, line 9 of Exercise 8: "$\tilde{I}\subseteq k(\mathbf{t})[\mathbf{x}]$" should be "$\tilde{I}\subseteq k[\mathbf{x},\mathbf{t}]$"

Page 316, part (a) of Exercise 8: Replace part (a) with the following:

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a. Fix j and suppose $g_j=\sum_{i=1}^s B_{ji}\,f_i$ in $k(\mathbf{t})[\mathbf{x}]$ and let $d_j\in k[\mathbf{t}]$ be a polynomial that clears the denominators of $B_{j1},\dots,B_{js}$. Also let $F=\operatorname{lcm}(F_1,\dots,F_s)$. Then prove that

$$d_j\in(\tilde{I}:F\tilde{g}_j)\cap k[\mathbf{t}],$$

where $\tilde{I}:F\tilde{g}_j$ the ideal quotient as defined in §4 of Chapter 4.

Page 316, part (b) of Exercise 8: "$(\tilde{I}:\tilde{g}_j)\cap k[\mathbf{t}]$" should be "$(\tilde{I}:F\tilde{g}_j)\cap k[\mathbf{t}]$"

Page 317, line 1: "a monomial on order" should be "a monomial order on"

Page 317, part (b) of Exercise 11: "$c_i\in\mathbb{R}[a,b,l_2,l_3]$" should be "$h_i\in\mathbb{R}[a,b,l_2,l_3]$"

Page 326, line 4 of the first display should be:

$$f_4 = x_3u_3 + x_4u_1 - x_4u_2 - u_1u_3,$$

Page 327, line 2 of **Definition 7**: "$\mathbb{R}[u_1, \ldots, u_m,\ x_1, \ldots, x_n]$" should be "$\mathbb{R}[u_1, \ldots, u_m, x_1, \ldots, x_n]$"

Page 331, part (b) of Exercise 2: Replace with "b. With this choice, explain why we can specify the coordinates of B as $B = (u_3, 0)$, i.e., the x-coordinate of B is arbitrary, but the y-coordinate is zero."

Page 332, Exercise 10: "made in Example 1" should be "made in the continuation of Example 1"

Page 332, line -1: "reducible components" should be "irreducible components"

Page 333, line 2 of part (e) of Exercise 14: "follows from part (a)" should be "follows from part (b)"

Page 333, line 2 of part (e) of Exercise 14: "$(c \cdot g)$'s" should be "$(c \cdot g)^s$"

Page 333, line 3 of part (c) of Exercise 15: "show that $\overline{c}$ has" should be "show that $c\overline{c}$ has"

Page 338, line -7: "degree in x_n are reduced" should be "degree in x_n is reduced"

Page 348, line -6: "$\text{LT}(\sigma_1\sigma)$" should be "$\text{LT}(\sigma_1\sigma_2)$"

Page 354, first display of Exercise 11: "$h_{j-i}(x_k, \ldots, x_n)$" should be "$h_{j-i}(x_j, \ldots, x_n)$"

Page 355, line 1 of the display in Exercise 15: "$+ (-1)^{j-1}\sigma_{k-1}x_i + (-1)^j\sigma_k =$" should be "$+(-1)^{j-1}\sigma_{j-1}x_i + (-1)^j\sigma_j =$"

Page 355, line 3 of Exercise 16: "coefficients $\mathbb{F}_2$" should be "coefficients in $\mathbb{F}_2$"

Page 355, display of Exercise 18: "$= s_j = \sigma_1 s_{j-1} +$" should be "$= s_j - \sigma_1 s_{j-1} +$"

Page 356, line 7: "every linear map" should be "every invertible linear map"

Page 361, second display of **Example 13**: Replace with the following:

$$x^i y^j = \begin{cases} x^{2m}y^{2l} = (x^2)^m(y^2)^l & \text{if } i, j \text{ are even} \\ x^{2m+1}y^{2l+1} = (x^2)^m(y^2)^l xy & \text{if } i, j \text{ are odd.} \end{cases}$$

Page 362, Exercise 6: "$k[x, y, z]^G$" should be "$\mathbb{R}[x, y, z]^G$" in part (d) and again in part (e)

Page 362, part (a) of Exercise 7: "$k[x, y, z]^G$" should be "$\mathbb{R}[x, y, z]^G$"

Page 363, second line of the first display: The third factor of g should be "$(x - y + z)$"

Page 363, part (b) of Exercise 7: "$k[x, y, z]^G$" should be "$\mathbb{R}[x, y, z]^G$" twice one line below the display, once two lines below the display, and once three lines below the display

Page 364, part (b) of Exercise 14: "Use the method of Exercise 13" should be "Use the method of Exercise 12"

Page 367, line −1: At the end of the display, "$\sum_{|\beta|=|G|} R_G(x^\beta)u^\beta$" should be "$\sum_{|\beta|=|G|} b_\beta R_G(x^\beta)u^\beta$"

Page 369, line −8: "$g(y_1, \ldots, y_m)$ for some $g \in k[y_1, \ldots, y_m]$" should be "$h(y_1, \ldots, y_m)$ for some $h \in k[y_1, \ldots, y_m]$"

Page 369, line −6: "$+\, g(y_1, \ldots, y_m)$" should be "$+\, h(y_1, \ldots, y_m)$"

Page 369, line −5: "g need not be" should be "h need not be"

Page 369, line −1: "divide g by G'" should be "divide h by G'"

Page 370, line 1: "$g = B_1 g_1 +$" should be "$h = B_1 g_1 +$"

Page 370, line 8: "Since G a Gröbner" should be "Since G is a Gröbner"

Page 372, part (b) of Exercise 8: "use Exercise 6 and §2" should be "use Exercise 6 and Example 13 of §2"

Page 375, line 3: "let G be a" should be "let $\mathcal{G}$ be a"

Page 375, line 4: "$G \cap k[y_1, \ldots, y_m]$" should be "$\mathcal{G} \cap k[y_1, \ldots, y_m]$"

Page 376, line −11: "Gröbner basis G" should be "Gröbner basis $\mathcal{G}$"

Page 376, line −11: "let $\overline{g}^G$ be the" should be "let $\overline{g}^{\mathcal{G}}$ be the"

Page 376, line −10: "on division by G" should be "on division by $\mathcal{G}$"

Page 376, line −10: "the remainders $\overline{g}^G$" should be "the remainders $\overline{g}^{\mathcal{G}}$"

Page 382, part (c) of Exercise 2: "Use Exercise 13" should be "Use Exercise 16"

Page 382, line 2 of Exercise 9: "$b = A \cdot \mathbf{a}$" should be "$\mathbf{b} = A \cdot \mathbf{a}$"

Page 383, line 2 of Exercise 12: "$G \cdot \mathbf{b} \cup G \cdot \mathbf{a} - \{\mathbf{a}\}$" should be '$(G \cdot \mathbf{b} \cup G \cdot \mathbf{a}) \setminus \{\mathbf{a}\}$"

Page 383, line 6 of Exercise 15: "chapter–then" should be "chapter—then"

Page 383, line 2 of Exercise 16: "as in Definition 1 of" should be "as in Definition 2 of"

Page 385, line −12: "*homogeneous coordinates*. to" should be "*homogeneous coordinates* to"

Page 385, line −12: "treatment of $\mathbb{P}^2(\mathbb{R})$ Our" should be "treatment of $\mathbb{P}^2(\mathbb{R})$. Our"

Page 394, Exercise 7: "the map (2)" should be "the map (1)"

Page 394, part (b) of Exercise 8: "in the from" should be "in the form"

Page 399, line −2: "$(1 : a_1 : \cdots : a_n)$" with "$\phi(a_1, \ldots, a_n) = (1 : a_1 : \cdots : a_n)$"

Page 400, line 16: "in some projective variety" should be "for some projective variety"

Page 404, line 5: "$1 \leq i_1 <$" should be "$0 \leq i_1 <$"

Page 404, line 2 of Exercise 9: "$f_i \in k[x_0, \ldots, x_n]$" should be "$f_j \in k[x_0, \ldots, x_n]$"

Page 409, line 5: "decomposed to" should be "decomposed into"

Page 410, line −1: "$k[x_1, \ldots, x_n]$" should be "$k[x_0, \ldots, x_n]$"

Page 413, part (a) of Exercise 3, lines 2 and 3: "$f = a_1 f_1 + \cdots + a_s f_s + r$. Prove that the quotients $a_1, \ldots, a_s$" should be "$f = q_1 f_1 + \cdots + q_s f_s + r$. Prove that the quotients $q_1, \ldots, q_s$"

Page 413, part (a) of Exercise 3, line 3: "remainder r" should be "remainder r"

Page 414, part (b) of Exercise 13: "$V \setminus V \cap \mathbf{V}(g)$" should be "$V \setminus (V \cap \mathbf{V}(g))$"

Page 416, line 2 of **Lemma 5**: "$\text{LM}_{>h}(f^h)$" should be "$\text{LM}_{>_h}(f^h)$"

Page 416, line −4: "$\text{LM}_{>h}(f^h)$" should be "$\text{LM}_{>_h}(f^h)$"

Page 417, equation (2): "$\text{LM}_{>h}(f^h)$" should be "$\text{LM}_{>_h}(f^h)$"

Page 417, two lines below equation (2): "$\text{LM}_{>h}(f^h)$" should be "$\text{LM}_{>_h}(f^h)$"

Page 419, second sentence of the proof of **Theorem 8**: The sentence should be "Applying the proof of part (i) of Proposition 7 with I in place of $\mathbf{I}_a(W)$ shows that Z is a projective variety containing W."

Page 423, line 16: "$\mathbb{P}^1(\mathbb{C}) \times \mathbb{C}$ and you" should be "$\mathbb{P}^1(\mathbb{C}) \times \mathbb{C}$, and you"

Page 425, first line following fourth display: "trivial solutions $(0{:}0; y)$" should be "trivial solutions $(0, 0, y)$"

Page 428, line −11: "Cramer's Rule" should be "Cramer's rule"

Page 429, line −1: "This proves $f \in I^{(0)} \cap \cdots \cap I^{(n)}$" should be "This proves $f \in I_n^{(0)} \cap \cdots \cap I_n^{(n)}$"

Page 430, line 17: "Now suppose $f \in I^{(i)}$" should be "Now suppose $f \in I_n^{(i)}$"

Page 430, line 20: "$f \in I^{(0)} \cap \cdots \cap I^{(n)}$" should be "$f \in I_n^{(0)} \cap \cdots \cap I_n^{(n)}$"

Page 431, line 2 of the proof of **Proposition 8**: "Then the proof of Proposition 7" should be 'Then Proposition 7"

Page 432, line 2 after the proof of **Proposition 10**: "$xy^2 - x + 1$ is a Gröbner basis" should be "$\{xy^2 - x + 1\}$ is a Gröbner basis"

Page 432, line -1: "point in $\mathbb{P}^n$" should be "point in $\mathbb{P}^m$"

Page 433, line 1 of the proof of **Theorem 11**: "The first has three parts" should be "The proof has three parts"

Page 433, line -12: "all have weight d" should be "are all weighted homogeneous of weight d"

Page 434, line -7: "there is a some" should be "there is some"

Page 443, line below display (9): "σ suppose that" should be "suppose that"

Page 445, first display: "$\begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_1 & b_3 \end{pmatrix}$" should be "$\begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & b_3 \end{pmatrix}$"

Page 446, line 9: "An straightforward" should be "A straightforward"

Page 446, line -3: "$w_{ij} = \lambda w'_{ij}$" should be "$w'_{ij} = \lambda w_{ij}$"

Page 447, line 5: "through two points" should be "through two distinct points"

Page 447, line 10: "are nonzero, and, hence, determine a line L" should be "are nonzero and distinct, and, hence, determine a unique line L"

Page 448, part (a) of Exercise 5: "$\sum_{i,j=0}^{n} a_{ij} x_i x_i$" should be "$\sum_{i,j=0}^{n} a_{ij} x_i x_j$"

Page 448, line 1 of Exercise 9: "be nonzero" should be "be nonzero with $Q = (a_{ij})$ symmetric"

Page 449, line 3 of Exercise 10: "set of all lines" should be "union of all projective lines"

Page 449, part (a) of Exercise 13: At the end of line 2, add "The image of F is called a *projective line* in $\mathbb{P}^n$."

Page 450, line 5: "$V \subseteq \mathbb{P}^4$" should be "$V \subseteq \mathbb{P}^9$"

Page 451, line 1: "lies is $\mathbf{V}(f)$" should be "lies in $\mathbf{V}(f)$"

Page 451, line 2: "all i" should be "for all i"

Page 451, line 4: "$p =$" should be "$u =$"

Page 451, line 5: "$f(p) = 1$" should be "$f(u) = c_i$"

Page 451, line 6: In two places, "$g(p)$" should be "$g(u)$"

Page 453, line -2: "nonzero polynomial" should be "nonconstant polynomial"

Page 454, line 15: "irreducible factors f" should be "irreducible factors of f"

Page 456, line 7: "$f = b_0 z^m + \cdots$" should be "$f = a_0 z^m + \cdots$"

Page 456, line 8: "$b_0 \in \mathbb{C} \setminus \{0\}$" should be "$a_0 \in \mathbb{C} \setminus \{0\}$"

Page 464, part (a) of Exercise 5: "nonzero polynomial" should be "nonconstant polynomial"

Page 472, line 4 of part (b) of Exercise 4: Add "(This is a challenging exercise.)"

Page 481, lines -7 and -6: "It is easy to generalize this argument and show" should be "By using the discussion following Lemma 4, one can show"

Page 495, part (c) of Exercise 10: "with Theorem 15 of Chapter 4, §3" should be "with Proposition 1 of §1"

Page 495, part (c) of Exercise 13: "Lemma 5 of §2" should be "Lemma 4 of §2"

Page 496, Exercise 7: "${}^aHF_I(s) = {}^aHP_I(s)$" should be "${}^aHF_{R/I}(s) = {}^aHP_{R/I}(s)$"

Page 505, part (a) of Exercise 11: "$\phi([f])$" should be "$\phi([f])$" (more space between [and f)

Page 505, part (a) of Exercise 11: "$(f(p_1)$," should be "$(f(p_1)$," (more space between (and f)

Page 506, part (a) of Exercise 14: "part (a) of the proposition" should be "part (i) of the proposition"

Page 506, part (b) of Exercise 14: "part (b) of the proposition" should be "part (ii) of the proposition"

Page 514, part (a) of Exercise 10: "If $f_1, \ldots, f_s \in k[x_1, \ldots, x_n]$" should be "If $f, f_1, \ldots, f_s \in k[x_1, \ldots, x_n]$"

Page 535, line -1: "$k \geq N$" should be "$i \geq N$"

Page 536, line 5: "$W \subset$" should be "$W \subseteq$"

Page 536, line 1 of Exercise 13: "$W \subset$" should be "$W \subseteq$"

Page 537, line 4 of part (d) of Exercise 14: "a curve $L \subseteq \Gamma$" should be "a curve $\widetilde{L} \subseteq \Gamma$"

Page 537, hint to part (a) of Exercise 15: "$\mathrm{BL}_0 V$" should be "$\mathrm{Bl}_0 V$"

Page 537, part (b) of Exercise 16: "$q(q, tq) = 0$" should be "$q(tq, q) = 0$"

Page 537, part (c) of Exercise 16: "$q(q, tq) = 0$" should be "$q(tq, q) = 0$"

Page 570, first display: On the left, "$\dfrac{\operatorname{lcm}(\operatorname{LM}(f_i), \operatorname{LM}(f_j)}{\operatorname{LT}(f_j)}$" should be "$\dfrac{\operatorname{lcm}(\operatorname{LM}(f_i), \operatorname{LM}(f_j))}{\operatorname{LT}(f_i)}$" (two errors)

Page 597, line -10: "Cramer's Rule" should be "Cramer's rule"