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## Ideals, Varieties and Algorithms, *fourth edition*

*Errata for the fourth edition as of August 18, 2017*

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Page 23, part (c) of Exercise 5: “Adapt the argument given at the end of the section” should be “Adapt the argument used for the circle  $x^2 + y^2 = 1$ ”

Page 24, line 1: “adapt the argument given at the end of the section” should be “adapt the argument used for the circle  $x^2 + y^2 = 1$ ”

Page 37, line 2 of part (c) of Exercise 16: “Binomial Theorem” should be “binomial theorem”

Page 39, line 6 of the paragraph beginning “To see why this algorithm works”: “By (5),” should be “By (1),”

Page 53, line 2 of Exercise 5: “affine algebraic varieties” should be “affine varieties”

Page 63, bottom display, second line underneath  $\overline{\quad}$ : “ $xy^2 - x$ ” should be “ $x^2y - x$ ”

Page 63, bottom display, fourth line underneath  $\overline{\quad}$ : “ $x^2y - y$ ” should be “ $xy^2 - y$ ”

Page 66, lines –5 and –6: “[using condition (ii) of the definition of a monomial order]” should be “[using Lemma 8 of §2]”

Page 81, line 1 of Exercise 2: “ $\text{LT}(I)$ ” should be “ $\langle \text{LT}(I) \rangle$ ”

Page 82, last line of Exercise 13: “Exercise 14 of Chapter 1, §4” should be “Proposition 8 of Chapter 1, §4”

Page 89, part (d) of Exercise 5: “ $z^2 - 3z$ ” should be “ $z^2 - 3z$ ”

Page 93, line –3: “ $\langle \text{LT}(G \setminus \{p\}) \rangle$ ” should be “ $\langle \text{LT}(G \setminus \{g\}) \rangle$ ”

Page 96, line 4 of part (b) of Exercise 10: “ $g_j = x_t + D$ ” should be “ $g_j = x_\ell + D$ ”

Page 96, line 4 of part (b) of Exercise 10: “ $S(g_i, g_j) = x_t C - x_s D$ ” should be “ $S(g_i, g_j) = x_\ell C - x_s D$ ”

Page 96, line 2 of part (a) of Exercise 12: “is not divisible by” should be “has leading term not divisible by”

Page 97, line 3 of Exercise 14: “ $\frac{x_j - a_j}{a_i - a_j}$ ” should be “ $\frac{x - a_j}{a_i - a_j}$ ”

Page 105, lines 13–16: Replace  $xy + 1$  with  $xy - 1$  in three places and  $-x - y$  with  $-x + y$  in two places. Thus the lines should be as follows:

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§3. If we divide  $f = xy^2 - x$  by  $G = (xy - 1, y^2 - 1)$ , the division algorithm gives

$$xy^2 - x = y \cdot (xy - 1) + 0 \cdot (y^2 - 1) + (-x + y)$$

so that  $\overline{f}^G = -x + y \neq 0$ . Yet we can also write

$$xy^2 - x = 0 \cdot (xy - 1) + x \cdot (y^2 - 1),$$

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Page 107, line -4: “ $f_3 = xz + y - z + 1$ ” should be “ $f_3 = xz - x + y + 1$ ”

Page 108, Exercise 1: “ $f_3 = xz + y - z + 1$ ” should be “ $f_3 = xz - x + y + 1$ ”

Page 110, Lemma 4: The statement of the lemma should be changed to the following:

**Lemma 4.** *Every element of  $S(F)$  can be written as a sum of homogeneous elements of  $S(F)$ . Furthermore, this decomposition is unique.*

Page 115, line 12 “by Lemma 2” should be “by Lemma 2 of §9”

Page 136, line 20: “ $(t, u, x, y, z) \in \mathbf{V}(I) \subseteq \mathbb{R}^5$ ” should be “ $(t, u, x, y, z) \in \mathbf{V}(I) \subseteq \mathbb{C}^5$ ”

Page 140, Exercise 3: “ $t^2$  is always positive” should be “ $t^2$  is always  $\geq 0$ ”

Page 142, part (a) of Exercise 11: In two places, replace  $F$  by  $G$

Page 142, part (b) of Exercise 11: Replace  $F$  by  $G$

Page 142, line 3 of Exercise 13: “ $x_i = f_i(t)/g_i(t)$ ” should be “ $x_\ell = f_\ell(t)/g_\ell(t)$ ”

Page 142, line 4 of Exercise 13: “each  $i$ ,  $f_i(t)$  and  $g_i(t)$ ” should be “each  $\ell$ ,  $f_\ell(t)$  and  $g_\ell(t)$ ”

Page 153, line 1: “straight lines” should be “straight line”

Page 153, line 4: “These lines have” should be “This line has”

Page 154, part (c) of Exercise 15: The  $x$ -coordinate of the second displayed point should be

$$\pm \frac{1}{2} \sqrt{15 + 6\sqrt[3]{2} - 12\sqrt[3]{4}}$$

Page 160, part (b) of Exercise 4: “ $g_o = g_3$ ” should be “ $g_o = g_2$ ”

Page 167, line -16: Replace “It follows that” with “(Lemma 3 and Proposition 5 apply to  $f, g$  since their coefficients lie in the field  $k(x_2, \dots, x_n)$ .) It follows that”

Page 169, line -7: “ $u_1(x_1)$ ” should be “ $u(x_1)$ ”

Page 170, line 1: “ $\langle f_i, f_* \rangle$ ” should be “ $\langle f_i, f^* \rangle$ ”

Page 170, part (c) of Exercise 3: “part (a) is still true but part (b) can fail” should be “parts (a) and (b) are still true”

Page 171, line 2 of Exercise 9: “ $l \geq m$ ” should be “ $l \geq m > 0$ ”

Page 180, line 13: “must have  $f_i(a_1, \dots, a_n) = 0$ ” should be “must have  $f_i(a_1, \dots, a_n) \neq 0$ ”

Page 181, lines 1 and 2 of Exercise 10: “ $\mathbb{R}[x, y]$ ” should be “ $\mathbb{R}[x, y, z]$ ” in two places.

Page 181, line 4 of Exercise 10: “same for  $\mathbb{R}[x]$ ” should be “same for  $\mathbb{R}[x]$  and  $\mathbb{R}[x, y]$ ”

Page 187, line -3: “ $a_1 \frac{\partial f_i}{\partial x_j} h_i$ ” should be “ $a_i \frac{\partial f_i}{\partial x_j} h_i$ ”

Page 189, Exercise 15: Replace the hint with “Hint: Show that  $xy, xz, yz$  generate the ideal of leading terms of  $\sqrt{I}$  and use the definition of Gröbner basis given in Chapter 1, §5.”

Page 190, line 15: “We have sketched  $\mathbf{V}(I)$  and  $\mathbf{V}(J)$  on the next page” should be “We have sketched  $\mathbf{V}(I)$  and  $\mathbf{V}(J)$  below”

Page 196, line 4: “principal ideals is principal)” should be “two principal ideals is principal)”

Page 199, lines 1–2 of **Definition 2**: “smallest affine algebraic variety” should be “smallest affine variety”

Page 201, line 12: “varieties  $k^n$ ” should be “varieties in  $k^n$ ”

Page 206, Exercise 7: “and the Exercise 4” should be “and Exercise 4”

Page 219, line -18: “is projection” should be “be the projection”

Page 221, **Corollary 3**: “With the same notation” should be “With  $k$  algebraically closed and the same notation”

Page 222, line 4: “for all  $i$ ” should be “for all such  $i$ ”

Page 222, line 6: “If follows that” should be “It follows that”

Page 222, line immediately before **Proposition 5**: “what works for  $V$ ” should be “that works for  $V$ ”

Page 222, line 4 of **Proposition 5**: “ $\pi_l(\mathbf{V}(I^{(i)}))$ ” should be “ $\pi_l(\mathbf{V}(I^{(i)}))$ ”

Page 222, line 5 of **Proposition 5**: “a variety contained in  $V$ ” should be “a variety contained in  $\mathbf{V}(I_l)$ ”

Page 223, line -17: “ $W \subsetneq \mathbf{V}(I)$ ” should be “ $W \subsetneq \mathbf{V}(I_l)$ ”

Page 223, line -9: “fails for  $I, \mathbf{V}(I) \setminus$ ” should be “fails for  $I, \mathbf{V}(I_l) \setminus$ ”

Page 223, line –8: “by Proposition 4” should be “by Proposition 4 (we can assume  $G$  is reduced)”

Page 224, second display: The display should be as follows:

$$\mathbf{V}(I_1) \setminus \mathbf{V}(c_1) = \mathbb{C} \setminus \mathbf{V}(y) = \mathbb{C} \setminus \{0\} \subseteq \pi_1(\mathbf{V}(I)) \subseteq \mathbf{V}(I_1) = \mathbb{C}.$$

Page 226, two lines below display (2): “ $W_2 \setminus Z_2 \subset \pi_l(V_1)$ ” should be “ $W_2 \setminus Z_2 \subseteq \pi_l(V_1)$ ”

Page 231, line 9: “**EXERCISES FOR §9**” should be “**EXERCISES FOR §8**”

Page 231, part (b) of Exercise 11: “Exercise 4” should be “Exercise 6”

Page 232, line 2: “The table on the next page” should be “The table below”

Page 247, line 1: “ $R = k[x, t]$ ” should be “ $R = k[t]$ ”

Page 247, line 2 of part (b) of Exercise 10: “ $a, b \in k[x]$ ” should be “ $a, b \in k$ ”

Page 255, line 12: “polynomials  $G_{i-1}$ ” should be “polynomials in  $G_{i-1}$ ”

Page 257, part (e) of Exercise 11: “we developed in Chapter 1” should be “we developed in Chapter 1, §2”

Page 260, lines 2–4: These three lines

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(iii) is proved in the same way as Theorem 11 of Chapter 4, §5. □

When  $k$  is algebraically closed, the Weak Nullstellensatz also holds in  $k[V]$ . You will prove this in Exercise 16.

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should be replaced with the following:

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(iii) is proved by first showing that the Weak Nullstellensatz also holds in  $k[V]$ . You will prove this in Exercise 16. From here, one proceeds in the same way as Theorem 11 of Chapter 4, §5. □

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Page 271, line following second display: “ $\mathbf{V}_W(a^2 - b^2 + 4)$ ” should be “ $\mathbf{V}_W(y^2 - z^2 + 4)$ ”

Page 272, second paragraph of the proof of **Proposition 6**: In two places, “ $\mathbf{V}(f_i g'_i - f'_i g_i)$ ” should be “ $\mathbf{V}_V(f_i g'_i - f'_i g_i)$ ”

Page 278, line –6: “since we do not the ideal” should be “since we do not want the ideal”

Page 279, display (1): Replace the display with

$$(1) \quad s^\ell + c_1 s^{\ell-1} + \cdots + c_\ell = 0, \quad c_1, \dots, c_\ell \in R.$$

Page 279, line –10: “ $a_{i\ell} s_{i\ell}$ ” should be “ $a_{i\ell} s_\ell$ ”

Page 279, line -7: “the coefficient of  $x$  is” should be “the coefficient of  $x^\ell$  is”

Page 279, the last display should be:

$$\det(A - xI_\ell) = (-1)^\ell(x^\ell + c_1x^{\ell-1} + \cdots + c_\ell).$$

Page 279, line -4: “ $a_i \in R$ ” should be “ $c_i \in R$ ”

Page 280, line -13: “(ii)  $\Rightarrow$  (iii)” should be “(i)  $\Rightarrow$  (iii)”

Page 280, line -9: “divide  $f$  by  $G$ ” should be “divide  $f$  by a Gröbner basis  $G$ ”

Page 282, seven lines below the figure: “the Zariski closure” should be “is the Zariski closure”

Page 282, line -6: “finite over  $k[y]$ ” should be “finite over  $k[y_1, \dots, y_m]$ ”

Page 284, line 1: “The surprise that” should be “The surprise is that”

Page 284, line 10: “algebraically independently” should be “algebraically independent”

Page 288, line 2 of Exercise 6: “means geometrically” should be “means geometrically when  $k$  is algebraically closed”

Page 289, last line of part (a) of Exercise 17: “ $\phi(\mathbf{V}(J))$ ” should be “ $\pi(\mathbf{V}(J))$ ”

Page 291, line 8: The correct hyphenation is “re-searchers”

Page 294, line below second display: “ $C = U \times V$ ” should be “ $\mathcal{C} = U \times V$ ”

Page 298, two lines below third display: The correct hyphenation is “ex-ercises”

Page 300, first display: “ $f(\theta_1 + \theta_2 + \theta_3)$ ” should be “ $f(\theta_1, \theta_2, \theta_3)$ ”

Page 300, paragraph beginning “We will next discuss”: Insert a blank line between the beginning of this paragraph and the end of the previous paragraph.

Page 302, line 2: “ $\mathcal{J} = \mathbf{V}(x_1^2 + y_1^2 - 1, x_2^2 + y_2^2 - 1, x_3^2 + y_3^2 - 1)$ ” should be “ $\mathcal{J} = \mathbf{V}(c_1^2 + s_1^2 - 1, c_2^2 + s_2^2 - 1, c_3^2 + s_3^2 - 1)$ ”

Page 302, two lines below (7): “ $V = \mathbf{V}(x_1^2 + y_1^2 - 1, x_2^2 + y_2^2 - 1, x_3^2 + y_3^2 - 1)$ ” should be “ $V = \mathbf{V}(c_1^2 + s_1^2 - 1, c_2^2 + s_2^2 - 1, c_3^2 + s_3^2 - 1)$ ”

Page 303, Exercise 2: The correct hyphenation is “co-ordinates”

Page 304, part (a) of Exercise 9: “result of part (c)” should be “result of part (e)”

Page 305, line 12: “in equation (7) of §2” should be “in equation (6) of §2”

Page 305, line 1 of (2): “ $\frac{2bl_2l_3}{2l_2(a^2 + b^2)}s_2$ ” should be “ $\frac{bl_2l_3}{l_2(a^2 + b^2)}s_2$ ”

Page 305, line 2 of (2): “ $\frac{2al_2l_3}{2l_2(a^2 + b^2)}s_2 +$ ” should be “ $\frac{al_2l_3}{l_2(a^2 + b^2)}s_2 -$ ”

Page 306, line 1 of (3): “ $\frac{2b}{2(a^2 + b^2)}s_2$ ” should be “ $\frac{b}{a^2 + b^2}s_2$ ”

Page 306, line 2 of (3): “ $\frac{2a}{2(a^2 + b^2)}s_2 +$ ” should be “ $\frac{a}{a^2 + b^2}s_2 -$ ”

Page 307, line –3: “when  $a^2 + b^2 < 4$ ,” should be “when  $0 < a^2 + b^2 < 4$ ,”

Page 308, line 6: “if  $l_4$  lies in” should be “if  $l_2 = l_3 = 1$  and  $l_4$  lies in”

Page 308, line 8 of the subsection *Specialization of Gröbner Bases*: “ $k[x_1, \dots, x_m, t_1, \dots, t_m]$ ” should be “ $k[x_1, \dots, x_n, t_1, \dots, t_m]$ ”

Page 309, display in the middle of the page: Replace the display with

$$1, l_2, l_3, l_2, l_3, 1, l_2l_3, l_2l_3, l_2l_3, a, b, a^2 + b^2, l_2l_3.$$

Page 309, two lines below display: “ $a, b, l_2, l_3, a^2 + b^2$  and  $a^2 + b^2 - l_2^2 - l_3^2$  are nonzero” should be “ $a, b, l_2, l_3$  and  $a^2 + b^2$  are nonzero”

Page 309, nine lines below display: “concept of a” should be “the concept of a”

Page 309, line –4: “to be Gröbner cover” should be “to be a Gröbner cover”

Page 312, line 3: “We have” should be “When  $l_2 = l_3 = 1$ , we have”

Page 315, part (c) of Exercise 5: “the choice of  $t \in \mathbb{R}$ .” should be “the choice of  $t \in \mathbb{R}?$ ”

Page 316, line 5: “ $B_{ij} \in k(\mathbf{t})[\mathbf{x}]$ ” should be “ $B_{ji} \in k(\mathbf{t})[\mathbf{x}]$ ”

Page 316, part (c) of Exercise 7: Replace the hint with “Hint: The monomial orders for  $k(\mathbf{t})[\mathbf{x}]$  and  $k[\mathbf{x}]$  are the same—the parameters  $t_j$  are “constants” as far as the ordering is concerned. Theorem 6 of Chapter 2, §9 will be useful.”

Page 316, lines 7 and 8 of Exercise 8: Delete these lines and replace them with the following:

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nonzero polynomials  $F_i$  and  $G_j$  in  $k[\mathbf{t}]$ , we get

$$\tilde{f}_i = F_i f_i, \tilde{g}_j = G_j g_j \in k[\mathbf{x}, \mathbf{t}].$$

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Page 316, line 9 of Exercise 8: “ $\tilde{I} \subseteq k(\mathbf{t})[\mathbf{x}]$ ” should be “ $\tilde{I} \subseteq k[\mathbf{x}, \mathbf{t}]$ ”

Page 316, part (a) of Exercise 8: Replace part (a) with the following:

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- a. Fix  $j$  and suppose  $g_j = \sum_{i=1}^s B_{ji} f_i$  in  $k(\mathbf{t})[\mathbf{x}]$  and let  $d_j \in k[\mathbf{t}]$  be a polynomial that clears the denominators of  $B_{j1}, \dots, B_{js}$ . Also let  $F = \text{lcm}(F_1, \dots, F_s)$ . Then prove that

$$d_j \in (\tilde{I} : F \tilde{g}_j) \cap k[\mathbf{t}],$$

where  $\tilde{I} : F \tilde{g}_j$  the ideal quotient as defined in §4 of Chapter 4.

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Page 316, part (b) of Exercise 8: “ $(\tilde{I} : \tilde{g}_j) \cap k[\mathbf{t}]$ ” should be “ $(\tilde{I} : F\tilde{g}_j) \cap k[\mathbf{t}]$ ”

Page 316, part (b) of Exercise 8: “to describe an algorithm for finding the subset  $W \subseteq k^n$  described in” should be “to give an algorithm for finding a subset  $W \subseteq k^n$  with the property described in”

Page 317, line 1: “a monomial on order” should be “a monomial order on”

Page 317, part (b) of Exercise 11: “ $c_i \in \mathbb{R}[a, b, l_2, l_3]$ ” should be “ $h_i \in \mathbb{R}[a, b, l_2, l_3]$ ”

Page 326, line 4 of the first display should be:

$$f_4 = x_3u_3 + x_4u_1 - x_4u_2 - u_1u_3,$$

Page 327, line 2 of **Definition 7**: “ $\mathbb{R}[u_1, \dots, u_m, x_1, \dots, x_n]$ ” should be “ $\mathbb{R}[u_1, \dots, u_m, x_1, \dots, x_n]$ ”

Page 331, part (b) of Exercise 2: Replace with “b. With this choice, explain why we can specify the coordinates of  $B$  as  $B = (u_3, 0)$ , i.e., the  $x$ -coordinate of  $B$  is arbitrary, but the  $y$ -coordinate is zero.”

Page 332, Exercise 10: “made in Example 1” should be “made in the continuation of Example 1”

Page 332, line -1: “reducible components” should be “irreducible components”

Page 333, line 2 of part (e) of Exercise 14: “follows from part (a)” should be “follows from part (b)”

Page 333, line 2 of part (e) of Exercise 14: “ $(c \cdot g)$ ’s” should be “ $(c \cdot g)^s$ ”

Page 333, line 3 of part (c) of Exercise 15: “show that  $\bar{c}$  has” should be “show that  $c\bar{c}$  has”

Page 336, lines 6-7: Interchange the order of these two lines. Thus the line “ $q := \dots$ ” should be above the lines “ $r := \dots$ ”

Page 338, line -7: “degree in  $x_n$  are reduced” should be “degree in  $x_n$  is reduced”

Page 348, line -6: “ $\text{LT}(\sigma_1\sigma)$ ” should be “ $\text{LT}(\sigma_1\sigma_2)$ ”

Page 354, first display of Exercise 11: “ $h_{j-i}(x_k, \dots, x_n)$ ” should be “ $h_{j-i}(x_j, \dots, x_n)$ ”

Page 355, line 1 of the display in Exercise 15: “ $+(-1)^{j-1}\sigma_{k-1}x_i + (-1)^j\sigma_k =$ ” should be “ $+(-1)^{j-1}\sigma_{j-1}x_i + (-1)^j\sigma_j =$ ”

Page 355, line 3 of Exercise 16: “coefficients  $\mathbb{F}_2$ ” should be “coefficients in  $\mathbb{F}_2$ ”

Page 355, display of Exercise 18: “ $= s_j = \sigma_1s_{j-1} +$ ” should be “ $= s_j - \sigma_1s_{j-1} +$ ”

Page 356, line 7: “every linear map” should be “every invertible linear map”

Page 361, second display of **Example 13**: Replace with the following:

$$x^i y^j = \begin{cases} x^{2m} y^{2l} = (x^2)^m (y^2)^l & \text{if } i, j \text{ are even} \\ x^{2m+1} y^{2l+1} = (x^2)^m (y^2)^l xy & \text{if } i, j \text{ are odd.} \end{cases}$$

Page 362, Exercise 6: “ $k[x, y, z]^G$ ” should be “ $\mathbb{R}[x, y, z]^G$ ” in part (d) and again in part (e)

Page 362, part (a) of Exercise 7: “ $k[x, y, z]^G$ ” should be “ $\mathbb{R}[x, y, z]^G$ ”

Page 363, second line of the first display: The third factor of  $g$  should be “ $(x - y + z)$ ”

Page 363, part (b) of Exercise 7: “ $k[x, y, z]^G$ ” should be “ $\mathbb{R}[x, y, z]^G$ ” twice one line below the display, once two lines below the display, and once three lines below the display

Page 364, part (b) of Exercise 14: “Use the method of Exercise 13” should be “Use the method of Exercise 12”

Page 367, line –1: At the end of the display, “ $\sum_{|\beta|=|G|} R_G(x^\beta)u^\beta$ ” should be “ $\sum_{|\beta|=|G|} b_\beta R_G(x^\beta)u^\beta$ ”

Page 369, line –8: “ $g(y_1, \dots, y_m)$  for some  $g \in k[y_1, \dots, y_m]$ ” should be “ $h(y_1, \dots, y_m)$  for some  $h \in k[y_1, \dots, y_m]$ ”

Page 369, line –6: “ $+g(y_1, \dots, y_m)$ ” should be “ $+h(y_1, \dots, y_m)$ ”

Page 369, line –5: “ $g$  need not be” should be “ $h$  need not be”

Page 369, line –1: “divide  $g$  by  $G$ ” should be “divide  $h$  by  $G$ ”

Page 370, line 1: “ $g = B_1 g_1 +$ ” should be “ $h = B_1 g_1 +$ ”

Page 370, line 8: “Since  $G$  a Gröbner” should be “Since  $G$  is a Gröbner”

Page 372, part (b) of Exercise 8: “use Exercise 6 and §2” should be “use Exercise 6 and Example 13 of §2”

Page 375, line 3: “let  $G$  be a” should be “let  $\mathcal{G}$  be a”

Page 375, line 4: “ $G \cap k[y_1, \dots, y_m]$ ” should be “ $\mathcal{G} \cap k[y_1, \dots, y_m]$ ”

Page 376, line –11: “Gröbner basis  $G$ ” should be “Gröbner basis  $\mathcal{G}$ ”

Page 376, line –11: “let  $\bar{g}^G$  be the” should be “let  $\bar{g}^{\mathcal{G}}$  be the”

Page 376, line –10: “on division by  $G$ ” should be “on division by  $\mathcal{G}$ ”

Page 376, line –10: “the remainders  $\bar{g}^G$ ” should be “the remainders  $\bar{g}^{\mathcal{G}}$ ”

Page 382, part (c) of Exercise 2: “Use Exercise 13” should be “Use Exercise 16”



Page 382, line 2 of Exercise 9: “ $b = A \cdot \mathbf{a}$ ” should be “ $\mathbf{b} = A \cdot \mathbf{a}$ ”

Page 383, line 2 of Exercise 12: “ $G \cdot \mathbf{b} \cup G \cdot \mathbf{a} - \{\mathbf{a}\}$ ” should be “ $(G \cdot \mathbf{b} \cup G \cdot \mathbf{a}) \setminus \{\mathbf{a}\}$ ”

Page 383, line 6 of Exercise 15: “chapter–then” should be “chapter—then”

Page 383, line 2 of Exercise 16: “as in Definition 1 of” should be “as in Definition 2 of”

Page 385, line –12: “*homogeneous coordinates. to*” should be “*homogeneous coordinates to*”

Page 385, line –12: “treatment of  $\mathbb{P}^2(\mathbb{R})$  Our” should be “treatment of  $\mathbb{P}^2(\mathbb{R})$ . Our”

Page 394, Exercise 7: “the map (2)” should be “the map (1)”

Page 394, part (b) of Exercise 8: “in the from” should be “in the form”

Page 399, line –2: “ $(1 : a_1 : \dots : a_n)$ ” with “ $\phi(a_1, \dots, a_n) = (1 : a_1 : \dots : a_n)$ ”

Page 400, line 16: “in some projective variety” should be “for some projective variety”

Page 404, line 5: “ $1 \leq i_1 <$ ” should be “ $0 \leq i_1 <$ ”

Page 404, line 2 of Exercise 9: “ $f_i \in k[x_0, \dots, x_n]$ ” should be “ $f_j \in k[x_0, \dots, x_n]$ ”

Page 409, line 5: “decomposed to” should be “decomposed into”

Page 410, line –1: “ $k[x_1, \dots, x_n]$ ” should be “ $k[x_0, \dots, x_n]$ ”

Page 413, part (a) of Exercise 3, lines 2 and 3: “ $f = a_1 f_1 + \dots + a_s f_s + r$ . Prove that the quotients  $a_1, \dots, a_s$ ” should be “ $f = q_1 f_1 + \dots + q_s f_s + r$ . Prove that the quotients  $q_1, \dots, q_s$ ”

Page 413, part (a) of Exercise 3, line 3: “remainder r” should be “remainder  $r$ ”

Page 414, part (b) of Exercise 13: “ $V \setminus V \cap \mathbf{V}(g)$ ” should be “ $V \setminus (V \cap \mathbf{V}(g))$ ”

Page 416, line 2 of **Lemma 5**: “ $\text{LM}_{>h}(f^h)$ ” should be “ $\text{LM}_{>_h}(f^h)$ ”

Page 416, line –4: “ $\text{LM}_{>h}(f^h)$ ” should be “ $\text{LM}_{>_h}(f^h)$ ”

Page 417, equation (2): “ $\text{LM}_{>h}(f^h)$ ” should be “ $\text{LM}_{>_h}(f^h)$ ”

Page 417, two lines below equation (2): “ $\text{LM}_{>h}(f^h)$ ” should be “ $\text{LM}_{>_h}(f^h)$ ”

Page 419, second sentence of the proof of **Theorem 8**: The sentence should be “Applying the proof of part (i) of Proposition 7 with  $I$  in place of  $\mathbf{I}_a(W)$  shows that  $Z$  is a projective variety containing  $W$ .”

Page 423, line 16: “ $\mathbb{P}^1(\mathbb{C}) \times \mathbb{C}$  and you” should be “ $\mathbb{P}^1(\mathbb{C}) \times \mathbb{C}$ , and you”

Page 425, first line following fourth display: “trivial solutions  $(0:0;y)$ ” should be “trivial solutions  $(0, 0, y)$ ”

Page 428, line –11: “Cramer’s Rule” should be “Cramer’s rule”

Page 429, line –1: “This proves  $f \in I^{(0)} \cap \dots \cap I^{(n)}$ ” should be “This proves  $f \in I_n^{(0)} \cap \dots \cap I_n^{(n)}$ ”

Page 430, line 17: “Now suppose  $f \in I^{(i)}$ ” should be “Now suppose  $f \in I_n^{(i)}$ ”

Page 430, line 20: “ $f \in I^{(0)} \cap \dots \cap I^{(n)}$ ” should be “ $f \in I_n^{(0)} \cap \dots \cap I_n^{(n)}$ ”

Page 431, line 2 of the proof of **Proposition 8**: “Then the proof of Proposition 7” should be “Then Proposition 7”

Page 432, line 2 after the proof of **Proposition 10**: “ $xy^2 - x + 1$  is a Gröbner basis” should be “ $\{xy^2 - x + 1\}$  is a Gröbner basis”

Page 432, line –1: “point in  $\mathbb{P}^n$ ” should be “point in  $\mathbb{P}^m$ ”

Page 433, line 1 of the proof of **Theorem 11**: “The first has three parts” should be “The proof has three parts”

Page 433, line –12: “all have weight  $d$ ” should be “are all weighted homogeneous of weight  $d$ ”

Page 434, line –7: “there is a some” should be “there is some”

Page 443, line below display (9): “ $\sigma$  suppose that” should be “suppose that”

Page 445, first display: “ $\begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_1 & b_3 \end{pmatrix}$ ” should be “ $\begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & b_3 \end{pmatrix}$ ”

Page 446, line 9: “An straightforward” should be “A straightforward”

Page 446, line –3: “ $w_{ij} = \lambda w'_{ij}$ ” should be “ $w'_{ij} = \lambda w_{ij}$ ”

Page 447, line 5: “through two points” should be “through two distinct points”

Page 447, line 10: “are nonzero, and, hence, determine a line  $L$ ” should be “are nonzero and distinct, and, hence, determine a unique line  $L$ ”

Page 448, part (a) of Exercise 5: “ $\sum_{i,j=0}^n a_{ij}x_i x_j$ ” should be “ $\sum_{i,j=0}^n a_{ij}x_i x_j$ ”

Page 448, line 1 of Exercise 9: “be nonzero” should be “be nonzero with  $Q = (a_{ij})$  symmetric”

Page 449, line 3 of Exercise 10: “set of all lines” should be “union of all projective lines”

Page 449, part (a) of Exercise 13: At the end of line 2, add “The image of  $F$  is called a *projective line* in  $\mathbb{P}^n$ .”

Page 450, line 5: “ $V \subseteq \mathbb{P}^4$ ” should be “ $V \subseteq \mathbb{P}^9$ ”

Page 451, line 1: “lies is  $\mathbf{V}(f)$ ” should be “lies in  $\mathbf{V}(f)$ ”

Page 451, line 2: “all  $i$ ” should be “for all  $i$ ”

Page 451, line 4: “ $p =$ ” should be “ $u =$ ”

Page 451, line 5: “ $f(p) = 1$ ” should be “ $f(u) = c_i$ ”

Page 451, line 6: In two places, “ $g(p)$ ” should be “ $g(u)$ ”

Page 453, line  $-2$ : “nonzero polynomial” should be “nonconstant polynomial”

Page 454, line 15: “irreducible factors  $f$ ” should be “irreducible factors of  $f$ ”

Page 456, line 7: “ $f = b_0z^m + \dots$ ” should be “ $f = a_0z^m + \dots$ ”

Page 456, line 8: “ $b_0 \in \mathbb{C} \setminus \{0\}$ ” should be “ $a_0 \in \mathbb{C} \setminus \{0\}$ ”

Page 464, part (a) of Exercise 5: “nonzero polynomial” should be “nonconstant polynomial”

Page 472, line 4 of part (b) of Exercise 4: Add “(This is a challenging exercise.)”

Page 481, lines  $-7$  and  $-6$ : “It is easy to generalize this argument and show” should be “By using the discussion following Lemma 4, one can show”

Page 495, part (c) of Exercise 10: “with Theorem 15 of Chapter 4, §3” should be “with Proposition 1 of §1”

Page 495, part (c) of Exercise 13: “Lemma 5 of §2” should be “Lemma 4 of §2”

Page 496, Exercise 7: “ ${}^aHF_I(s) = {}^aHP_I(s)$ ” should be “ ${}^aHF_{R/I}(s) = {}^aHP_{R/I}(s)$ ”

Page 499, line 1: “*an algebraically closed*” should be “*an algebraically closed field*”

Page 505, part (a) of Exercise 11: “ $\phi([f])$ ” should be “ $\phi([f])$ ” (more space between [ and  $f$ )

Page 505, part (a) of Exercise 11: “ $(f(p_1),$ ” should be “ $(f(p_1),$ ” (more space between ( and  $f$ )

Page 506, part (a) of Exercise 14: “part (a) of the proposition” should be “part (i) of the proposition”

Page 506, part (b) of Exercise 14: “part (b) of the proposition” should be “part (ii) of the proposition”

Page 514, part (a) of Exercise 10: “If  $f_1, \dots, f_s \in k[x_1, \dots, x_n]$ ” should be “If  $f, f_1, \dots, f_s \in k[x_1, \dots, x_n]$ ”

Page 535, line -1: “ $k \geq N$ ” should be “ $i \geq N$ ”

Page 536, line 5: “ $W \subset$ ” should be “ $W \subseteq$ ”

Page 536, line 1 of Exercise 13: “ $W \subset$ ” should be “ $W \subseteq$ ”

Page 537, line 4 of part (d) of Exercise 14: “a curve  $L \subseteq \Gamma$ ” should be “a curve  $\tilde{L} \subseteq \Gamma$ ”

Page 537, hint to part (a) of Exercise 15: “ $\text{BL}_0V$ ” should be “ $\text{Bl}_0V$ ”

Page 537, part (b) of Exercise 16: “ $q(q, tq) = 0$ ” should be “ $q(tq, q) = 0$ ”

Page 537, part (c) of Exercise 16: “ $q(q, tq) = 0$ ” should be “ $q(tq, q) = 0$ ”

Page 549, part (a) of Exercise 7: “matrix  $M_3$  in (3)” should be “matrix  $M_3$  in (5)”

Page 549, line 2 of part (e) of Exercise 7: The line should be “ $x > y > z$  and explain its relation to the matrix  $N_2$  in Example 8.”

Page 559, line 21: “ $HF_{S/\langle \text{LT}(G) \rangle}(m') < HF_{S/I}(m')$ ” should be “ $HF_{S/\langle \text{LT}(G) \rangle}(m') > HF_{S/I}(m')$ ”

Page 564, line 3: “ $-v^2 + \xi^2 - \zeta^2$ ” should be “ $v^2 - \xi^2 + \zeta^2$ ”

Page 564, line 10: “ $-u\eta^2 + v\zeta^2$ ” should be “ $u\eta^2 - v\zeta^2$ ”

Page 564, line 12: “ $-u\xi^2 + u\zeta^2 + v\eta^2$ ” should be “ $u\xi^2 - u\zeta^2 - v\eta^2$ ”

Page 564, fourth display: “ $-\xi^2\zeta^2 + \eta^4 + \zeta^4$ ” should be “ $\xi^2\zeta^2 - \eta^4 - \zeta^4$ ”

Page 564, sixth display: “ $-xz + y^2 + z^2$ ” should be “ $xz - y^2 - z^2$ ”

Page 570, first display: On the left, “ $\frac{\text{lcm}(\text{LM}(f_i), \text{LM}(f_j))}{\text{LT}(f_j)}$ ” should be “ $\frac{\text{lcm}(\text{LM}(f_i), \text{LM}(f_j))}{\text{LT}(f_i)}$ ”,  
(two errors)

Page 578, line 3 of first display: “ $xy$ ” should be “ $-xy$ ”

Page 578, line -1: in two places, “ $(xy)$ ” should be “ $(-xy)$ ”

Page 579, line before last display: “ $-xy + y^2$ ” should be “ $xy + y^2$ ”

Page 582, line 2 of **Example 6**: “ $f_2 = xy^2 - xy$ ” should be “ $f_2 = xy^2 + xy$ ”

Page 582, first display of **Example 6**: “ $-x(xy^2 - xy)$ ” should be “ $-x(xy^2 + xy)$ ”

Page 582, line after first display of **Example 6**: “equal to  $\text{LT}(f_1)$ ” should be “equal to  $-\text{LT}(f_1)$ ”

Page 582, third display: “ $\mathfrak{s}(\mathbf{g}) = x\mathbf{e}_2$ ” should be “ $\mathfrak{s}(\mathbf{g}) = -x\mathbf{e}_2$ ”

Page 585, line above the second display: “ $y\mathbf{e}_1 - x\mathbf{e}_1$ ” should be “ $y\mathbf{e}_1 - x\mathbf{e}_2$ ”

Page 585, line below the second display: “ $\mathfrak{s}(\mathbf{h}) = x^2 \mathbf{e}_2$  divides  $\mathfrak{s}(\mathbf{k})$ ” should be “ $\mathfrak{s}(\mathbf{k}) = x^2 \mathbf{e}_2$  divides  $\mathfrak{s}(\mathbf{h})$ ”

Page 585, line below the second display: “a a” should be “a”

Page 586, first line of pseudocode: “ $f_i \in R$ ” should be “ $f_i \in R, f_i$  monic”

Page 590, part (c) of Exercise 1: On the first line, “order on  $S$ ” should be “order on  $R$ ”

Page 590, part (c) of Exercise 1: On the second line, “ $>$ ” should be “ $>_{POT}$ ”

Page 590, part (b) of Exercise 2: Replace with “If we allow  $\mathfrak{s}$ -reductions and reduction by the syzygy  $\mathbf{h} = (-f_2, f_1)$ , then show that we can reduce  $\mathbf{g}$  to  $(0, 0)$ .”

Page 590, part (c) of Exercise 2: Replace with “Use Propositions 12 and 14 to explain why the computations in parts (a) and (b) are unnecessary.”

Page 591, line 10 of the pseudocode in Exercise 5: “ $>$ ” should be “ $>_{POT}$ ”

Page 597, line -10: “Cramer’s Rule” should be “Cramer’s rule”